

6.4 Sampling and estimation

Name: _____ Class: _____ Date: _____

Total: 13 marks

KEY WORDS sample 样本 • confidence interval 置信区间 • population 总体 • central limit theorem 中心极限定理
• sample mean 样本均值

Objective

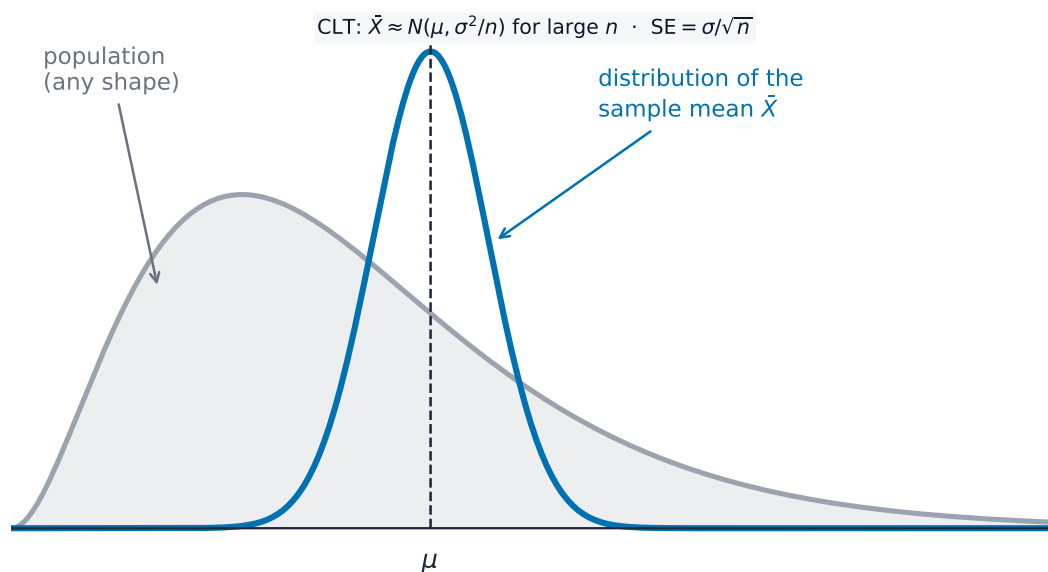
Build the skills to answer exam questions on **sampling and estimation** 抽样与估计 — the distribution of the **sample mean** 样本均值, the **Central Limit Theorem** 中心极限定理, **unbiased estimates** 无偏估计, and **confidence intervals** 置信区间.

You must be able to:

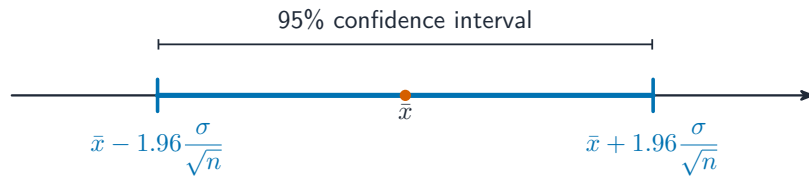
- use $E(\bar{X}) = \mu$, $\text{Var}(\bar{X}) = \frac{\sigma^2}{n}$ and the CLT
- find unbiased estimates of the population mean and variance
- find a confidence interval $\bar{x} \pm z \frac{\sigma}{\sqrt{n}}$

1 Worked examples

■ Confidence interval



By the CLT, $\bar{X}$ is approximately normal for large n , centred on μ with variance σ^2/n



A 95% interval reaches 1.96 standard errors each side of $\bar{x}$

$$n = 64, \bar{x} = 50, \sigma = 8: 50 \pm 1.96 \frac{8}{8} = 50 \pm 1.96 = (48.0, 52.0).$$

2 Practice

2.1 A population has $\sigma = 10$. For a sample of $n = 25$, find the standard error $\frac{\sigma}{\sqrt{n}}$. [2]

2.2 State the value of z used in a 95% confidence interval. [1]

3 Exam-style questions

3.1 As the sample size n increases, $\text{Var}(\bar{X}) = \frac{\sigma^2}{n}$: [1]

- **A** increases
- **B** decreases
- **C** stays the same
- **D** becomes negative

3.2 A random sample of 100 light bulbs has mean lifetime $\bar{x} = 1200$ hours. The population standard deviation is $\sigma = 150$ hours.

(a) Find a 95% confidence interval for the mean lifetime. [3]

(b) State what changes to the interval if a 99% level ($z = 2.576$) is used instead. [2]

3.3 A sample of 8 values gives $\sum x = 120$ and $\sum x^2 = 1840$. Find unbiased estimates of the population mean and variance. [4]

Solutions

2.1 $\frac{10}{\sqrt{25}} = \frac{10}{5} = 2.$

2.2 1.96.

3.1 B. Dividing by a larger n makes $\text{Var}(\bar{X})$ smaller.

3.2 (a) $1200 \pm 1.96 \frac{150}{\sqrt{100}} = 1200 \pm 1.96(15) = 1200 \pm 29.4; (1170.6, 1229.4).$

(b) the interval becomes **wider** (a higher confidence level needs a larger z , here 2.576).

3.3 mean $\bar{x} = \frac{120}{8} = 15$; unbiased variance $s^2 = \frac{1}{n-1} \left(\sum x^2 - \frac{(\sum x)^2}{n} \right) = \frac{1}{7} \left(1840 - \frac{120^2}{8} \right) = \frac{1}{7}(1840 - 1800) = \frac{40}{7} = 5.71.$