

5.5 The normal distribution

Name: _____ Class: _____ Date: _____

Total: 15 marks

KEY WORDS approximation 近似 • continuity correction 连续性校正 • normal distribution 正态分布
• standardisation 标准化 • standard deviation 标准差

Objective

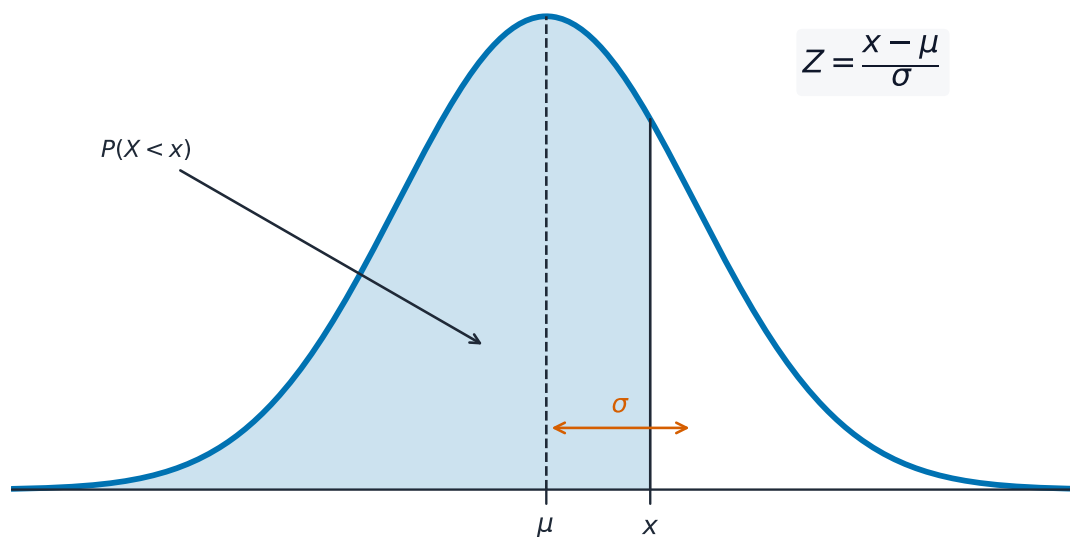
Build the skills to answer exam questions on the **normal distribution** 正态分布 — **standardisation** 标准化, reading the Φ table, finding an unknown μ or σ , and the **normal approximation to the binomial** 二项分布的正态近似.

You must be able to:

- standardise with $Z = \frac{X - \mu}{\sigma}$ and read Φ
- find an unknown mean or standard deviation from a given probability
- use the normal approximation with a **continuity correction** 连续性校正

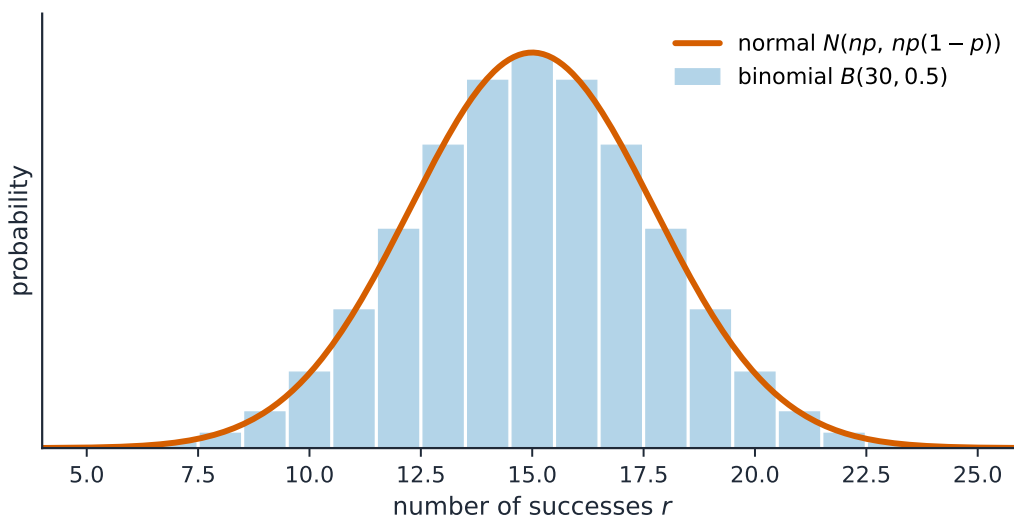
1 Worked examples

- Standardise & find μ



A normal probability is an area under the bell curve; standardising rescales it to $Z \sim N(0, 1)$

$X \sim N(\mu, 0.14^2)$, $P(X < 1.48) = 0.22$. $\Phi^{-1}(0.22) = -0.772$, so $\frac{1.48 - \mu}{0.14} = -0.772 \Rightarrow \mu = 1.59$.



approx $N(np, np(1-p))$ when n large · continuity correction

For large n the binomial $\approx N(np, np(1-p))$ — add a ± 0.5 continuity correction

2 Practice

2.1 $X \sim N(50, 4^2)$. Find the Z value for $X = 56$. [2]

2.2 State the continuity correction used for $P(X \geq 20)$ in a normal approximation. [1]

3 Exam-style questions

3.1 $X \sim N(0, 1)$. Using $\Phi(1) = 0.8413$, $P(Z > 1)$ is: [1]

- A 0.8413
- B 0.1587
- C 0.3413
- D 0.5

3.2 The heights of plants are $X \sim N(82, 5^2)$ cm.

(a) Find $P(X > 90)$. [3]

(b) Find the height exceeded by the tallest 10% of plants. [3]

3.3 A fair coin is tossed 100 times. Using a normal approximation, find the probability of obtaining at least 60 heads. [5]

Solutions

2.1 $Z = \frac{56 - 50}{4} = 1.5.$

2.2 use $P(X \geq 19.5).$

3.1 B. $P(Z > 1) = 1 - 0.8413 = 0.1587.$

3.2 (a) $Z = \frac{90 - 82}{5} = 1.6; P(X > 90) = 1 - \Phi(1.6) = 1 - 0.9452 = 0.0548.$

(b) top 10%: $\Phi(z) = 0.90 \Rightarrow z = 1.282; x = 82 + 1.282(5) = 88.4 \text{ cm}.$

3.3 $X \sim B(100, 0.5) \approx N(50, 25);$ continuity correction $P(X \geq 60) \rightarrow P(Y > 59.5);$

$Z = \frac{59.5 - 50}{5} = 1.9; P = 1 - \Phi(1.9) = 1 - 0.9713 = 0.0287.$