A-LEVEL MATHEMATICS • EXERCISE SHEET

4.1 Forces and equilibrium

Name: _____ Class: _____ Date: _____ **Total: 14 marks**

KEY WORDS friction 摩擦 • coefficient of friction 摩擦系数 • equilibrium 平衡 • forces and equilibrium 力与平衡 • limiting friction 最大静摩擦力

Objective

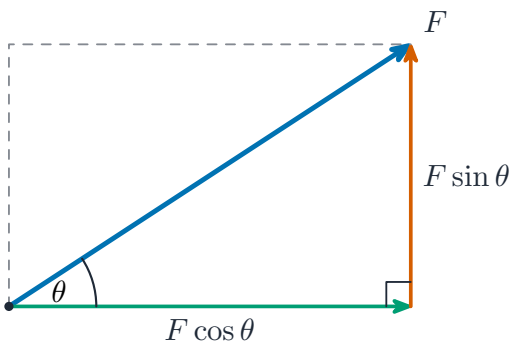
Build the skills to answer exam questions on **forces and equilibrium** 力与平衡 — resolving forces into **components** 分量, the condition for **equilibrium** 平衡, and **friction** 摩擦 ($F \leqslant \mu R$). Take $g = 10$ m s^{-2}.

You must be able to:

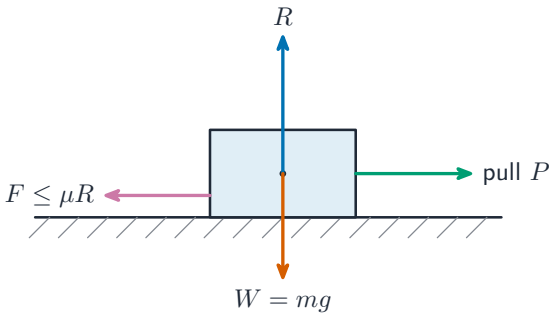
- resolve a force into horizontal and vertical components
- use "components in any direction sum to zero" for equilibrium
- use $F \leqslant \mu R$ and limiting friction $F = \mu R$

1 Worked examples

■ Resolving & friction



A force at angle θ has parts $F\cos\theta$ (horizontal) and $F\sin\theta$ (vertical)



The contact force splits into the normal reaction R and friction $F \leqslant \mu R$

A 5 kg block rests on a rough horizontal floor ($\mu = 0.3$). Max friction $= \mu R = 0.3 \times 50 = 15$ N — a horizontal pull below 15 N will not move it.

2 Practice

2.1 A force of 20 N acts at 60° above the horizontal. Find its horizontal component. [2]

2.2 A block of weight 40 N rests on a rough surface, $\mu = 0.25$. Find the maximum friction available. [2]

3 Exam-style questions

3.1 A particle is in equilibrium under three forces. The sum of the forces is: [1]

- **A** maximum
- **B** zero
- **C** equal to the weight
- **D** equal to μR

3.2 A box of mass 8 kg rests on a rough horizontal floor. A horizontal force of 30 N is applied but the box does not move.

(a) Find the normal reaction. [1]

(b) Given the box is on the point of slipping, find the coefficient of friction. [3]

3.3 A particle of weight W hangs in equilibrium from two light strings making angles 30° and 60° with the horizontal. The tensions are T_1 and T_2. Given $W = 20$ N, find T_1 and T_2. [5]

Solutions

2.1 $20\cos 60° = 20 \times 0.5 = 10$ N.

2.2 $F_{\max} = \mu R = 0.25 \times 40 = 10$ N.

3.1 **B**. In equilibrium the resultant (vector sum) of the forces is zero.

3.2 (a) $R = mg = 8 \times 10 = 80$ N.
(b) on the point of slipping $F = \mu R$ and $F = 30$ N; $\mu = \dfrac{30}{80}$; $= 0.375$.

3.3 resolve: horizontal $T_2\cos 60° = T_1\cos 30°$; vertical $T_1\sin 30° + T_2\sin 60° = 20$; from the first $T_2(0.5) = T_1(0.866) \Rightarrow T_2 = 1.732\,T_1$; substitute: $0.5T_1 + 0.866(1.732T_1) = 20 \Rightarrow 2T_1 = 20 \Rightarrow T_1 = 10$ N, $T_2 = 17.3$ N.