A-LEVEL MATHEMATICS • EXERCISE SHEET

# 3.9 Complex numbers

Name: _____ Class: _____ Date: _____ **Total: 14 marks**

**KEY WORDS** complex numbers 复数 • argand diagram 阿干图 • argument 辐角 • loci 轨迹 • modulus 模

## Objective

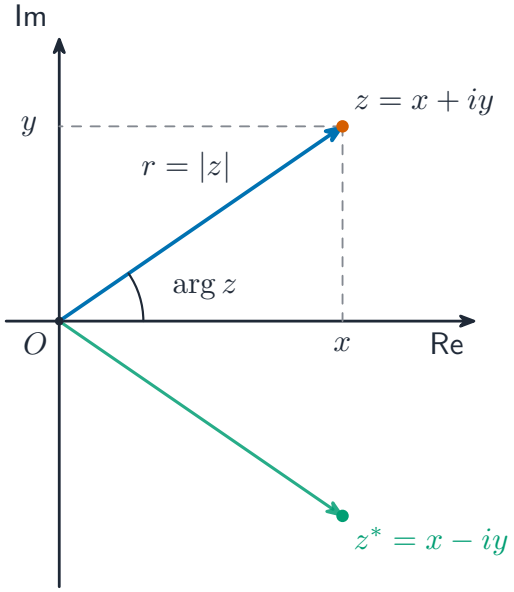
Build the skills to answer exam questions on **complex numbers** 复数 — arithmetic and **conjugates** 共轭, **modulus** 模 and **argument** 辐角, the **Argand diagram** 阿干图, and **loci** 轨迹.

**You must be able to:**

- add, multiply and divide complex numbers (divide using the conjugate)
- find modulus and argument; convert to polar form
- sketch loci such as $|z-a|=r$, $|z-a|=|z-b|$, $\arg(z-a)=\theta$

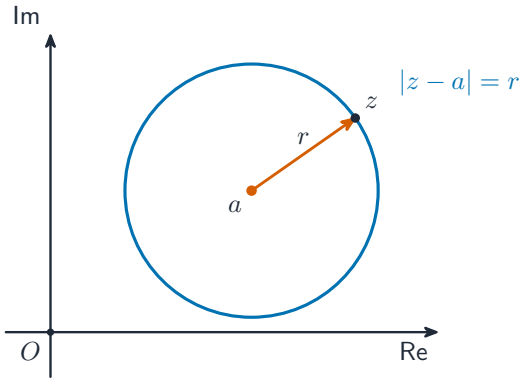
## 1 Worked examples

### ■ Division & loci




*$|z|$ is the distance from $O$, $\arg z$ the angle, $z^*$ the reflection in the real axis*

$$\frac{3+i}{1-i} = \frac{(3+i)(1+i)}{2} = \frac{2+4i}{2} = 1+2i.$$




*$|z-a|=r$ is a circle of radius $r$ centred at $a$*

## 2 Practice

**2.1** Find the modulus of $z = 3 + 4i$. [1]

**2.2** Write $(2+i)(3-i)$ in the form $x+iy$. [2]

## 3 Exam-style questions

**3.1** The argument of $z = -1 + i$ (in radians, $-\pi < \arg z \leqslant \pi$) is: [1]

- **A** $\frac{\pi}{4}$
- **B** $\frac{3\pi}{4}$
- **C** $-\frac{\pi}{4}$
- **D** $\frac{\pi}{2}$

**3.2** The complex number $z$ satisfies $z = \dfrac{10}{2-i}$.

(a) Express $z$ in the form $x+iy$. [3]

(b) Find $|z|$ and $\arg z$ (in radians). [3]

**3.3** On a single Argand diagram, sketch the locus of points $z$ for which:

(a) $|z - (2 + i)| = 2$. [2]

(b) Describe the locus $|z - 3| = |z + 1|$. [2]

## Solutions

**2.1** $|z| = \sqrt{3^2 + 4^2} = 5$.

**2.2** $(2 + i)(3 - i) = 6 - 2i + 3i - i^2 = 7 + i$.

**3.1 B**. The point $(-1, 1)$ is in the second quadrant: $\arg z = \pi - \frac{\pi}{4} = \frac{3\pi}{4}$.

**3.2** (a) $z = \dfrac{10(2 + i)}{(2 - i)(2 + i)} = \dfrac{20 + 10i}{5} = 4 + 2i$.
(b) $|z| = \sqrt{4^2 + 2^2} = \sqrt{20} = 2\sqrt{5}$; $\arg z = \tan^{-1} \frac{2}{4} = 0.4636$ rad.

**3.3** (a) a circle of radius 2 centred at the point $(2, 1)$.
(b) the perpendicular bisector of the segment joining 3 and $-1$ — i.e. the vertical line $x = 1$.