# 3.8 Differential equations

Name: ____________ Class: ________ Date: __________ **Total: 15 marks**

**KEY WORDS** general solution 通解 • initial condition 初始条件 • particular solution 特解 • differential equations 微
• separable 可分离变量

## Objective

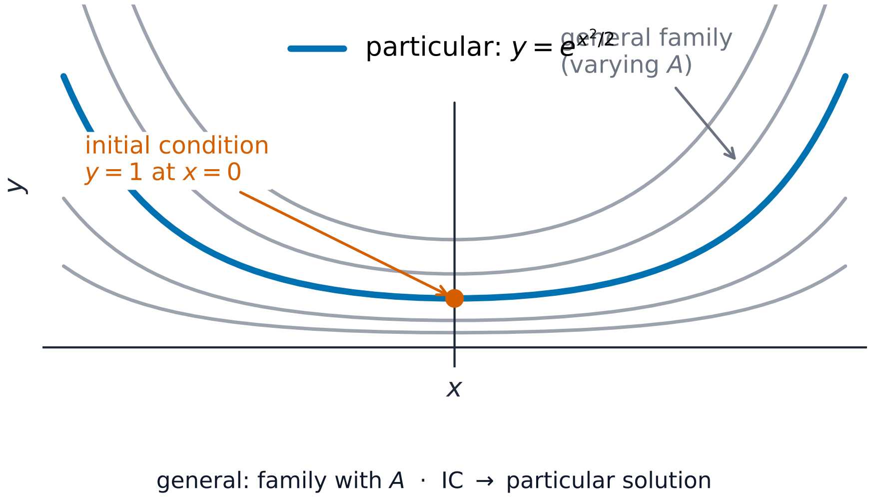
Build the skills to answer exam questions on **differential equations** 微分方程 — solving a **separable** 可分离变量 first-order equation, finding the **general** 通解 and **particular solution** 特解, and setting up a model.

**You must be able to:**

- separate variables and integrate both sides
- use an initial condition to find the constant
- form and solve a differential equation from a rate description

## 1 Worked examples

### ■ Separate and solve




*The constant gives a family of curves; an initial condition picks one particular solution*

**Solve $\frac{dy}{dx} = xy$, $y = 1$ at $x = 0$.**

$$\int \frac{1}{y}\,dy = \int x\,dx \Rightarrow \ln y = \tfrac{1}{2}x^2 + c \Rightarrow y = Ae^{x^2/2}; \quad y(0) = 1 \Rightarrow A = 1, \text{ so } y = e^{x^2/2}.$$

## 2 Practice

**2.1** Separate the variables for $\frac{dy}{dx} = \frac{y}{x}$ (write each side ready to integrate). [1]

**2.2** Solve $\frac{dy}{dx} = 3y$, giving the general solution. [2]

## 3 Exam-style questions

**3.1** The general solution of $\frac{dy}{dx} = ky$ is: [1]

- **A** $y = kx + c$
- **B** $y = Ae^{kx}$
- **C** $y = Ax^k$
- **D** $y = \frac{1}{2}kx^2 + c$

**3.2** Solve $\dfrac{dy}{dx} = 2x(y+1)$, given $y = 0$ when $x = 0$. Give $y$ explicitly. [5]

**3.3** A tank loses water so that the depth $h$ falls at a rate proportional to $\sqrt{h}$: $\dfrac{dh}{dt} = -k\sqrt{h}$. Initially $h = H$.

(a) Solve to show $\sqrt{h} = \sqrt{H} - \frac{1}{2}kt$. [4]

(b) Find, in terms of $H$ and $k$, the time for the tank to empty. [2]

## Solutions

**2.1** $\frac{1}{y}\,dy = \frac{1}{x}\,dx$.

**2.2** $\ln y = 3x + c \Rightarrow y = Ae^{3x}$.

**3.1** **B**. $\frac{dy}{dx} = ky$ gives $y = Ae^{kx}$.

**3.2** $\int \frac{1}{y+1}\,dy = \int 2x\,dx$; $\ln(y+1) = x^2 + c$; $y(0) = 0 \Rightarrow c = 0$; $y + 1 = e^{x^2}$; $y = e^{x^2} - 1$.

**3.3** (a) $\int h^{-1/2}\,dh = \int -k\,dt$; $2\sqrt{h} = -kt + c$; $h = H$ at $t = 0 \Rightarrow c = 2\sqrt{H}$; $\sqrt{h} = \sqrt{H} - \frac{1}{2}kt$.

(b) empty when $h = 0$: $0 = \sqrt{H} - \frac{1}{2}kt \Rightarrow t = \frac{2\sqrt{H}}{k}$.