

3.7 Vectors

Name: _____ Class: _____ Date: _____

Total: 16 marks

KEY WORDS vectors 向量 • position vector 位置向量 • scalar product 数量积 • scalar 标量 • magnitude 模长

Objective

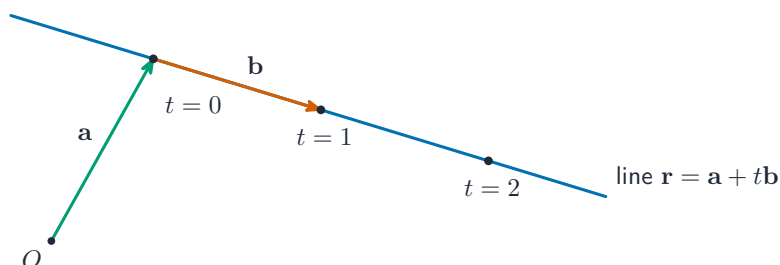
Build the skills to answer exam questions on **vectors** 向量 — magnitude and unit vectors, the **vector equation of a line** 直线向量方程, the **scalar product** 数量积, and the angle between lines.

You must be able to:

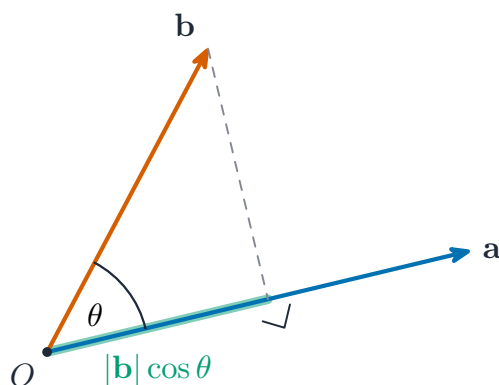
- find magnitude, a unit vector and a displacement $\vec{AB} = \mathbf{b} - \mathbf{a}$
- write and use $\mathbf{r} = \mathbf{a} + t\mathbf{b}$; test for intersection
- use $\mathbf{a} \cdot \mathbf{b} = |\mathbf{a}||\mathbf{b}| \cos \theta$ to find angles / perpendicularity

1 Worked examples

■ Line and scalar product



$\mathbf{r} = \mathbf{a} + t\mathbf{b}$: start at $\mathbf{a}$, add t copies of the direction $\mathbf{b}$



$\mathbf{a} \cdot \mathbf{b} = |\mathbf{a}||\mathbf{b}| \cos \theta$; it is 0 when the vectors are perpendicular

Angle between $\mathbf{a} = \mathbf{i} + 2\mathbf{j} + 2\mathbf{k}$, $\mathbf{b} = 2\mathbf{i} + 2\mathbf{j} + \mathbf{k}$: $\mathbf{a} \cdot \mathbf{b} = 8$, $|\mathbf{a}| = |\mathbf{b}| = 3$, so $\cos \theta = \frac{8}{9}$, $\theta = 27.3^\circ$.

2 Practice

2.1 Find $|\mathbf{v}|$ for $\mathbf{v} = 3\mathbf{i} - 4\mathbf{j} + 12\mathbf{k}$. [2]

2.2 Find $\mathbf{a} \cdot \mathbf{b}$ for $\mathbf{a} = 2\mathbf{i} - \mathbf{j} + \mathbf{k}$, $\mathbf{b} = \mathbf{i} + 3\mathbf{j} + \mathbf{k}$. [2]

3 Exam-style questions

3.1 Two vectors are perpendicular when their scalar product is: [1]

- **A** 1
 - **B** 0
 - **C** the product of their magnitudes
 - **D** undefined
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3.2 The line l has equation $\mathbf{r} = \begin{pmatrix} 1 \\ 0 \\ 2 \end{pmatrix} + t \begin{pmatrix} 2 \\ 1 \\ -1 \end{pmatrix}$.

(a) Find the position vector of the point on l where $t = 3$. [2]

(b) Find the acute angle between l and the direction $\mathbf{d} = \begin{pmatrix} 1 \\ 2 \\ 2 \end{pmatrix}$. [4]

3.3 Points $A(1, 2, 1)$, $B(3, 3, 4)$ and $C(2, 0, 3)$ are given.

(a) Find $\overrightarrow{AB}$ and $\overrightarrow{AC}$. [2]

(b) Find angle BAC . [3]

Solutions

2.1 $|\mathbf{v}| = \sqrt{9 + 16 + 144} = \sqrt{169} = 13.$

2.2 $\mathbf{a} \cdot \mathbf{b} = 2 - 3 + 1 = 0.$

3.1 B. Perpendicular vectors have scalar product 0.

3.2 (a) $\begin{pmatrix} 1 \\ 0 \\ 2 \end{pmatrix} + 3 \begin{pmatrix} 2 \\ 1 \\ -1 \end{pmatrix} = \begin{pmatrix} 7 \\ 3 \\ -1 \end{pmatrix}.$

(b) direction of l is $(2, 1, -1)$; $\mathbf{b} \cdot \mathbf{d} = 2 + 2 - 2 = 2$; $|\mathbf{b}| = \sqrt{6}$, $|\mathbf{d}| = 3$; $\cos \theta = \frac{2}{3\sqrt{6}} = 0.2722 \Rightarrow \theta = 74.2^\circ.$

3.3 (a) $\overrightarrow{AB} = (2, 1, 3)$, $\overrightarrow{AC} = (1, -2, 2).$

(b) $\overrightarrow{AB} \cdot \overrightarrow{AC} = 2 - 2 + 6 = 6$; $|\overrightarrow{AB}| = \sqrt{14}$, $|\overrightarrow{AC}| = 3$; $\cos(BAC) = \frac{6}{3\sqrt{14}} = 0.5345 \Rightarrow BAC = 57.7^\circ.$