

3.4 Differentiation (inverse tangent, implicit, parametric)

Name: _____ Class: _____ Date: _____

Total: 14 marks

KEY WORDS quotient 商 • inverse tangent 反正切 • differentiation 微分

Objective

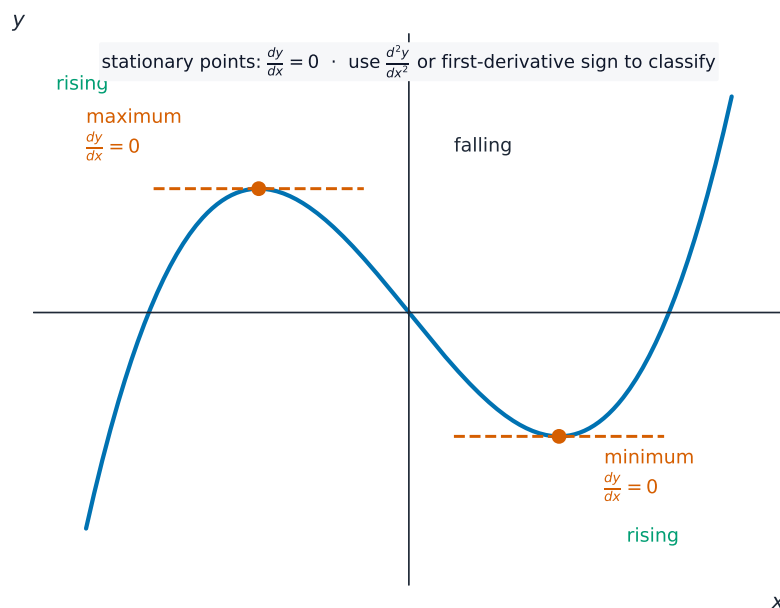
Build the skills to answer exam questions on **differentiation** 微分 in Pure 3 — the Pure 2 rules plus the **inverse tangent** 反正切 derivative, applied to parametric and implicit curves.

You must be able to:

- use $\frac{d}{dx} \tan^{-1} x = \frac{1}{1+x^2}$
- apply the product, quotient and chain rules
- differentiate parametric and implicit relations

1 Worked examples

- Mixed rules



Stationary points and gradients still come from differentiating — now with $\tan^{-1}$ and implicit methods too

$y = \tan^{-1}(2x)$: chain rule $\rightarrow \frac{dy}{dx} = \frac{1}{1 + (2x)^2} \cdot 2 = \frac{2}{1 + 4x^2}$.

Implicit $e^{2y} = x + y$: $2e^{2y} \frac{dy}{dx} = 1 + \frac{dy}{dx} \Rightarrow \frac{dy}{dx} = \frac{1}{2e^{2y} - 1}$.

2 Practice

2.1 Differentiate $y = \tan^{-1}(x^2)$. [2]

2.2 Differentiate $y = x \ln x$. [2]

3 Exam-style questions

3.1 $\frac{d}{dx} \tan^{-1}(3x)$ equals: [1]

- **A** $\frac{3}{1+9x^2}$
 - **B** $\frac{1}{1+3x^2}$
 - **C** $\frac{3}{1+3x^2}$
 - **D** $\frac{1}{1+9x^2}$
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3.2 A curve has parametric equations $x = 1 + 2 \cos t$, $y = 3 \sin t$ for $0 \leq t \leq \pi$. Find $\frac{dy}{dx}$ in terms of t and the gradient at $t = \frac{\pi}{6}$. [4]

3.3 A curve is given implicitly by $x^2y + y^3 = 10$.

(a) Find $\frac{dy}{dx}$ in terms of x and y . [3]

(b) Find the gradient at the point $(1, 2)$. [2]

Solutions

$$\mathbf{2.1} \quad \frac{dy}{dx} = \frac{2x}{1+x^4}.$$

$$\mathbf{2.2} \quad \frac{dy}{dx} = \ln x + 1.$$

$$\mathbf{3.1} \text{ A. } \frac{1}{1+(3x)^2} \cdot 3 = \frac{3}{1+9x^2}.$$

$$\mathbf{3.2} \quad \frac{dx}{dt} = -2 \sin t, \quad \frac{dy}{dt} = 3 \cos t; \quad \frac{dy}{dx} = \frac{3 \cos t}{-2 \sin t} = -\frac{3}{2} \cot t; \quad \text{at } t = \frac{\pi}{6}: \quad -\frac{3}{2} \cot 30^\circ = -\frac{3}{2} \sqrt{3} = -\frac{3\sqrt{3}}{2}.$$

$$\mathbf{3.3} \text{ (a) } 2xy + x^2 \frac{dy}{dx} + 3y^2 \frac{dy}{dx} = 0; \quad \frac{dy}{dx} = -\frac{2xy}{x^2 + 3y^2}.$$

$$\text{(b) at } (1, 2): \quad -\frac{2(1)(2)}{1+12} = -\frac{4}{13}.$$