

3.3 Trigonometry (identities, R-formula)

Name: _____ Class: _____ Date: _____

Total: 13 marks

KEY WORDS r-formula 辅助角公式 • trigonometry 三角学

Objective

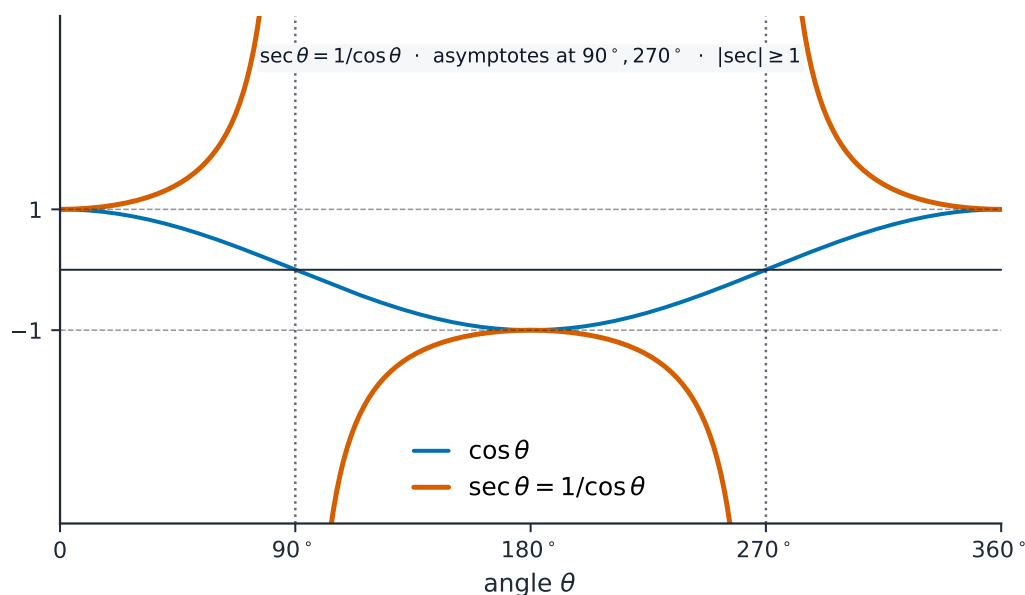
Build the skills to answer exam questions on **trigonometry** 三角学 in Pure 3 — the reciprocal identities, compound- and double-angle formulae, and the **R-formula** 辅助角公式, used in proofs and equations.

You must be able to:

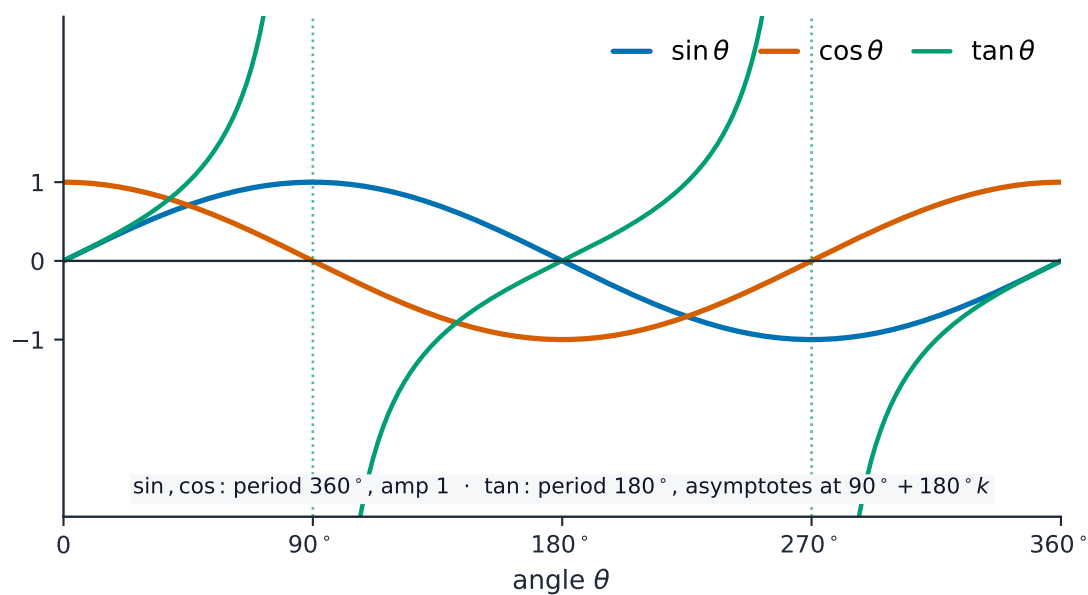
- prove identities using $\sec^2 \theta \equiv 1 + \tan^2 \theta$ etc.
- expand and use compound/double-angle formulae
- express $a \sin \theta + b \cos \theta$ as $R \sin(\theta \pm \alpha)$ and solve

1 Worked examples

■ R-formula in an equation



The reciprocal functions and the identities let you reduce an equation to one function



The R-formula rewrites a sum as a single sinusoid

Solve $3 \sin \theta + 4 \cos \theta = 2$ **for** $0^\circ \leq \theta \leq 360^\circ$. Write LHS as $5 \sin(\theta + 53.13^\circ)$: $\sin(\theta + 53.13^\circ) = 0.4$, so $\theta + 53.13^\circ = 23.6^\circ$ or $156.4^\circ (+360^\circ)$. Taking those in range gives $\theta = 103.3^\circ$ or 330.5° .

2 Practice

2.1 Write $\cos 2\theta$ in terms of $\cos \theta$ only.

[1]

2.2 Find R for $7 \sin \theta + 24 \cos \theta = R \sin(\theta + \alpha)$.

[1]

3 Exam-style questions

3.1 Using $\sin 2\theta = 2 \sin \theta \cos \theta$, the value of $\sin 2\theta$ when $\sin \theta = \frac{3}{5}$ and $\cos \theta = \frac{4}{5}$ is: [1]

- A $\frac{12}{25}$
 - B $\frac{24}{25}$
 - C $\frac{7}{25}$
 - D $\frac{6}{5}$
-

3.2 (a) Express $4 \sin \theta - 3 \cos \theta$ in the form $R \sin(\theta - \alpha)$, with $R > 0$ and $0^\circ < \alpha < 90^\circ$. [3]

(b) Hence solve $4 \sin \theta - 3 \cos \theta = 2$ for $0^\circ \leq \theta \leq 360^\circ$. [4]

3.3 Prove the identity $\frac{\sin 2\theta}{1 + \cos 2\theta} \equiv \tan \theta$. [3]

Solutions

2.1 $\cos 2\theta = 2 \cos^2 \theta - 1.$

2.2 $R = \sqrt{7^2 + 24^2} = 25.$

3.1 B. $2 \times \frac{3}{5} \times \frac{4}{5} = \frac{24}{25}.$

3.2 (a) $R = \sqrt{4^2 + 3^2} = 5; \tan \alpha = \frac{3}{4} \Rightarrow \alpha = 36.9^\circ; 5 \sin(\theta - 36.9^\circ).$

(b) $\sin(\theta - 36.9^\circ) = 0.4 \Rightarrow \theta - 36.9^\circ = 23.6^\circ \text{ or } 156.4^\circ; \theta = 60.5^\circ \text{ or } 193.3^\circ.$

3.3 $\sin 2\theta = 2 \sin \theta \cos \theta$ and $1 + \cos 2\theta = 2 \cos^2 \theta$; ratio $= \frac{2 \sin \theta \cos \theta}{2 \cos^2 \theta} = \frac{\sin \theta}{\cos \theta} = \tan \theta.$