3.1 Algebra (partial fractions, rational-power binomial)

Name: ______________ Class: __________ Date: ____________ **Total: 17 marks**

KEY WORDS partial fractions 部分分式 • binomial expansion 二项展开式 • further algebra 代数 • rational functior • converges 收敛

Objective

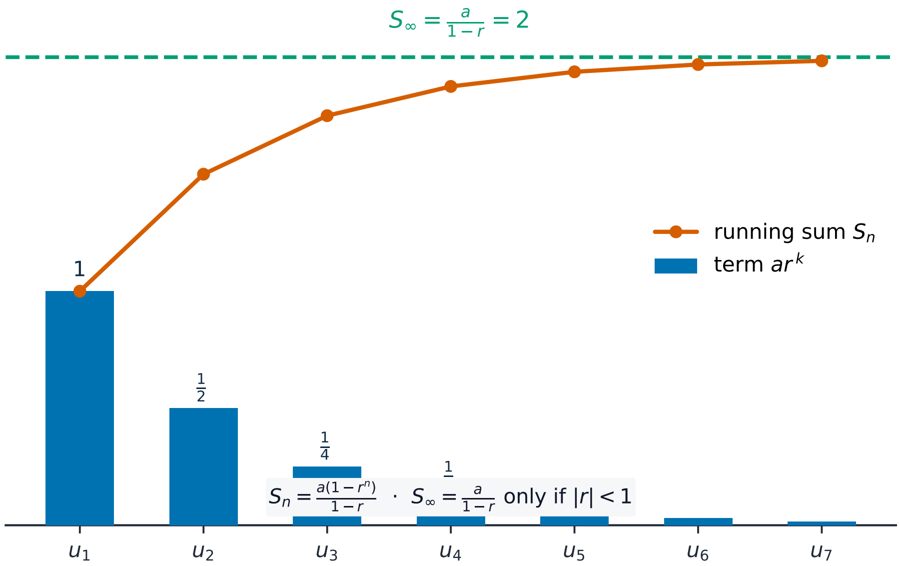
Build the skills to answer exam questions on **further algebra 代数 — partial fractions 部分分式** and the **binomial expansion for a rational power 分数次幂二项展开式**.

You must be able to:

- split a rational function into partial fractions (including a repeated factor)
- expand $(1 + x)^n$ for fractional/negative n (for $|x| < 1$)
- combine the two (expand after splitting)

1 Worked examples

■ Partial fractions & rational-power binomial



The rational-power binomial series converges only for $|x| < 1$ — the terms must shrink

$\dfrac{x+4}{(x+1)(x-2)} = \dfrac{A}{x+1} + \dfrac{B}{x-2}$: $x + 4 = A(x-2) + B(x+1)$; $x = 2 \Rightarrow B = 2$; $x = -1 \Rightarrow A = -1$. So $\dfrac{2}{x-2} - \dfrac{1}{x+1}$.

$(1+x)^{1/2} = 1 + \frac{1}{2}x - \frac{1}{8}x^2 + \cdots$ for $|x| < 1$.

2 Practice

2.1 Find the first three terms of $(1+x)^{-1}$. [2]

2.2 Write $\dfrac{5}{(x-1)(x+4)}$ in the form $\dfrac{A}{x-1} + \dfrac{B}{x+4}$. [3]

3 Exam-style questions

3.1 The expansion of $(1+2x)^{1/2}$ in ascending powers of x begins: [1]

- **A** $1+x-\frac{1}{2}x^2+\cdots$
- **B** $1+x+\frac{1}{2}x^2+\cdots$
- **C** $1+2x-2x^2+\cdots$
- **D** $1-x+x^2+\cdots$

3.2 Express $\dfrac{9x+5}{(x+1)(2x-1)}$ in partial fractions. [4]

3.3 (a) Express $\dfrac{2}{(1-x)(1+2x)}$ in partial fractions. [3]

(b) Hence find the expansion of $\dfrac{2}{(1-x)(1+2x)}$ up to the term in x^2. [4]

Solutions

2.1 $(1+x)^{-1} = 1 - x + x^2 - \cdots$.

2.2 $5 = A(x+4) + B(x-1)$; $x = 1 \Rightarrow A = 1$; $x = -4 \Rightarrow B = -1$; $\dfrac{1}{x-1} - \dfrac{1}{x+4}$.

3.1 **A**. $(1+2x)^{1/2} = 1 + \frac{1}{2}(2x) + \frac{(1/2)(-1/2)}{2}(2x)^2 + \cdots = 1 + x - \frac{1}{2}x^2 + \cdots$.

3.2 $9x + 5 = A(2x-1) + B(x+1)$; $x = -1 \Rightarrow -4 = -3A \Rightarrow A = \frac{4}{3}$; $x = \frac{1}{2} \Rightarrow 9.5 = 1.5B \Rightarrow B = \frac{19}{3}$; $\dfrac{4}{3(x+1)} + \dfrac{19}{3(2x-1)}$.

3.3 (a) $2 = A(1+2x) + B(1-x)$; $x = 1 \Rightarrow A = \frac{2}{3}$; $x = -\frac{1}{2} \Rightarrow B = \frac{4}{3}$; $\dfrac{2/3}{1-x} + \dfrac{4/3}{1+2x}$.
(b) $\frac{2}{3}(1 + x + x^2) + \frac{4}{3}(1 - 2x + 4x^2)$; $= \frac{2}{3} + \frac{4}{3} + (\frac{2}{3} - \frac{8}{3})x + (\frac{2}{3} + \frac{16}{3})x^2$; $= 2 - 2x + 6x^2$.