

2.5 Integration (standard forms, trapezium rule)

Name: _____ Class: _____ Date: _____

Total: 13 marks

KEY WORDS trapezium rule 梯形法则 • integration 积分

Objective

Build the skills to answer exam questions on **integration** 积分 — integrating e^{ax+b} , $\frac{1}{ax+b}$ and trig functions, powers of sin/cos via identities, and the **trapezium rule** 梯形法则.

You must be able to:

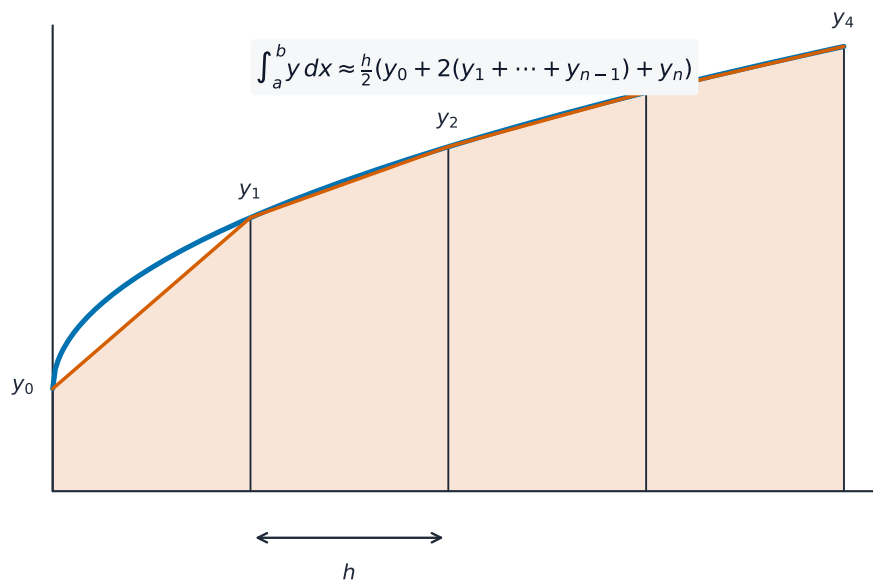
- integrate e^{ax+b} , $\frac{1}{ax+b}$, $\sin(ax+b)$, $\cos(ax+b)$, $\sec^2(ax+b)$
- integrate $\sin^2/\cos^2$ using a double-angle identity
- estimate a definite integral with the trapezium rule

1 Worked examples

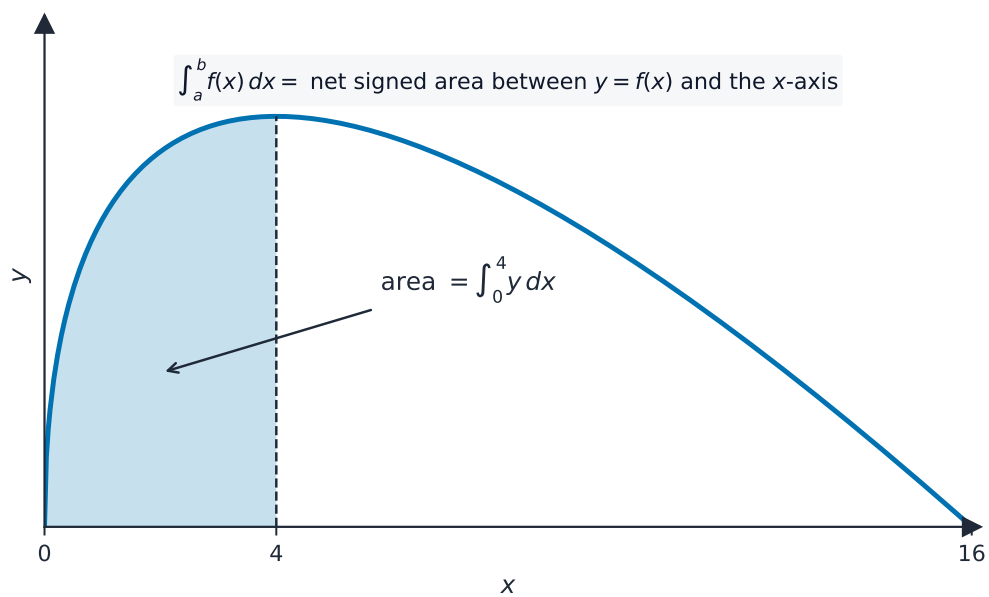
■ Powers of sin & the trapezium rule

$\int 6 \sin^2 x \, dx$ — use $\sin^2 x = \frac{1}{2}(1 - \cos 2x)$:

$$\int (3 - 3 \cos 2x) \, dx = 3x - \frac{3}{2} \sin 2x + C.$$



Each strip is a trapezium; their areas estimate

$$\int_a^b y dx \approx \frac{h}{2}[y_0 + y_n + 2(y_1 + \dots + y_{n-1})]$$


The trapezium rule estimates this area under the curve

2 Practice

2.1 Find $\int e^{2x+1} dx$. [2]

2.2 Find $\int \frac{1}{2x+3} dx$. [2]

3 Exam-style questions

3.1 $\int \cos 4x dx$ equals: [1]

- **A** $4 \sin 4x + C$
- **B** $\frac{1}{4} \sin 4x + C$
- **C** $-\frac{1}{4} \sin 4x + C$
- **D** $-4 \sin 4x + C$

3.2 Find $\int_0^{\pi/4} 4 \cos^2 x dx$, giving an exact answer. [4]

3.3 Use the trapezium rule with **four** strips to estimate $\int_0^2 \sqrt{1+x^3} dx$, giving your answer to 3 s.f. [4]

Solutions

2.1 $\frac{1}{2}e^{2x+1} + C.$

2.2 $\frac{1}{2} \ln |2x + 3| + C.$

3.1 B. $\int \cos 4x \, dx = \frac{1}{4} \sin 4x + C.$

3.2 $4 \cos^2 x = 2(1 + \cos 2x); \int_0^{\pi/4} (2 + 2 \cos 2x) \, dx = [2x + \sin 2x]_0^{\pi/4}; = \left(\frac{\pi}{2} + 1\right) - 0 = \frac{\pi}{2} + 1.$

3.3 $h = 0.5; y \text{ at } x = 0, 0.5, 1, 1.5, 2: 1, \sqrt{1.125} = 1.0607, \sqrt{2} = 1.4142, \sqrt{4.375} = 2.0917, \sqrt{9} = 3; \int \approx \frac{0.5}{2}[1 + 3 + 2(1.0607 + 1.4142 + 2.0917)]; = 0.25[4 + 2(4.5666)] = 0.25(13.133) = 3.28 \text{ (3 s.f.)}.$