2.4 Differentiation (product, quotient, parametric, implicit)

Name: ______________ Class: __________ Date: ____________ **Total: 14 marks**

KEY WORDS parametric 参数 • parametric equations 参数方程 • product 乘积法则 • implicit 隐式 • quotient rule 商法则

Objective

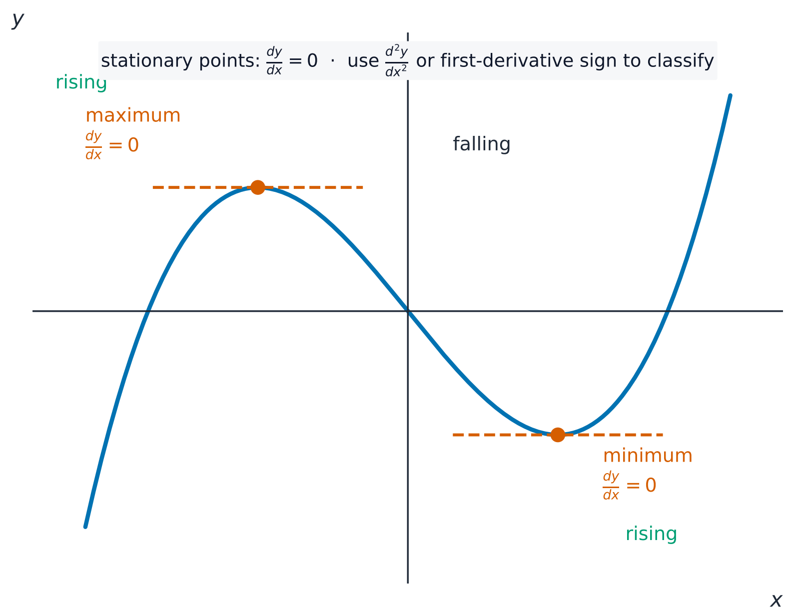
Build the skills to answer exam questions on **differentiation 微分** — standard derivatives (e^x, $\ln x$, trig), the **product 乘积法则** and **quotient rules 商法则**, and **parametric 参数** and **implicit 隐式** differentiation.

You must be able to:

- differentiate e^x, $\ln x$, $\sin x$, $\cos x$, $\tan x$
- apply the product and quotient rules
- differentiate parametric and implicit relations

1 Worked examples

■ Product rule & implicit



A stationary point ($\frac{dy}{dx}=0$) is still found by differentiating — now with the product/quotient/chain rules

$y = 6x\cos(x^2+1)$. Product rule ($u = 6x$, $v = \cos(x^2+1)$):

$$\frac{dy}{dx} = 6\cos(x^2+1) - 12x^2\sin(x^2+1).$$

Implicit: for $x^2+y^2=25$, differentiate: $2x + 2y\dfrac{dy}{dx} = 0 \Rightarrow \dfrac{dy}{dx} = -\dfrac{x}{y}$.

2 Practice

2.1 Differentiate $y = x^2e^x$. [2]

2.2 Differentiate $y = \dfrac{\sin x}{x}$. [2]

3 Exam-style questions

3.1 $\dfrac{d}{dx}(\ln(3x))$ equals: [1]

- **A** $\dfrac{1}{3x}$
- **B** $\dfrac{3}{x}$
- **C** $\dfrac{1}{x}$
- **D** $3\ln x$

3.2 A curve has parametric equations $x = t^2$, $y = t^3 - 3t$. Find $\dfrac{dy}{dx}$ in terms of t, and the value of t at the stationary point(s). [4]

3.3 A curve is defined implicitly by $x^2 + xy + y^2 = 7$.

(a) Find $\dfrac{dy}{dx}$ in terms of x and y. [3]

(b) Find the gradient of the curve at the point $(1, 2)$. [2]

Solutions

2.1 $\dfrac{dy}{dx} = 2xe^x + x^2e^x = xe^x(2+x)$.

2.2 $\dfrac{dy}{dx} = \dfrac{x\cos x - \sin x}{x^2}$.

3.1 **C**. $\ln(3x) = \ln 3 + \ln x$, so the derivative is $\dfrac{1}{x}$.

3.2 $\dfrac{dx}{dt} = 2t$, $\dfrac{dy}{dt} = 3t^2 - 3$; $\dfrac{dy}{dx} = \dfrac{3t^2-3}{2t}$; stationary where $3t^2 - 3 = 0 \Rightarrow t = \pm 1$.

3.3 (a) differentiate: $2x + y + x\dfrac{dy}{dx} + 2y\dfrac{dy}{dx} = 0$; $\dfrac{dy}{dx} = -\dfrac{2x+y}{x+2y}$.
(b) at $(1, 2)$: $\dfrac{dy}{dx} = -\dfrac{2+2}{1+4} = -\dfrac{4}{5}$.