

2.3 Trigonometry (reciprocal, compound, double angle, R-formula)

Name: _____ Class: _____ Date: _____

Total: 12 marks

KEY WORDS r-formula 辅助角公式 • compound 复合角 • double angle 二倍角 • further trigonometry 三角学

Objective

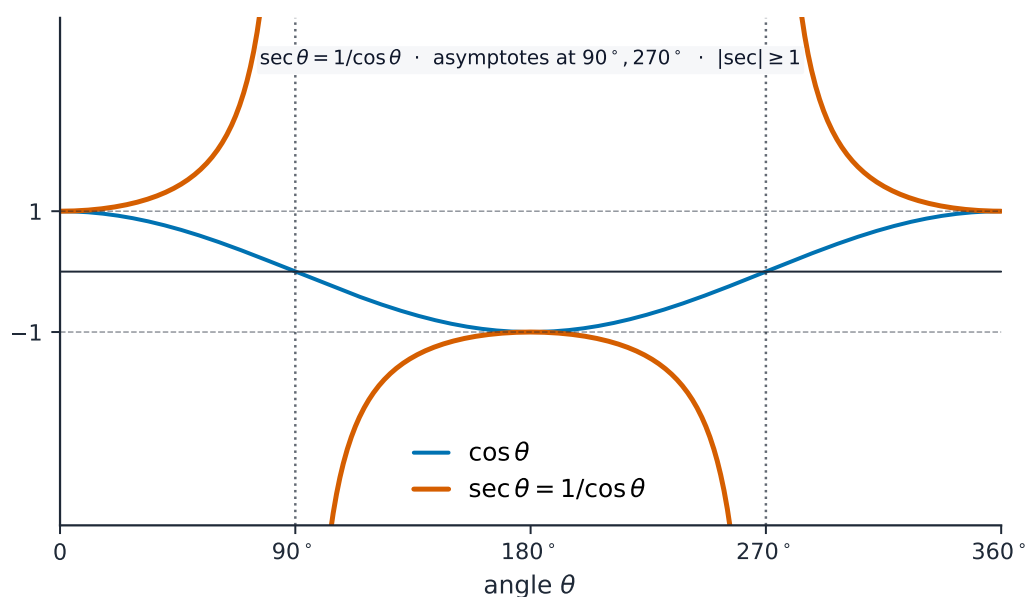
Build the skills to answer exam questions on **further trigonometry** 三角学 — the reciprocal functions, the identities $\sec^2 \theta \equiv 1 + \tan^2 \theta$, **compound** 复合角 and **double angle** 二倍角 formulae, and the **R-formula** 辅助角公式.

You must be able to:

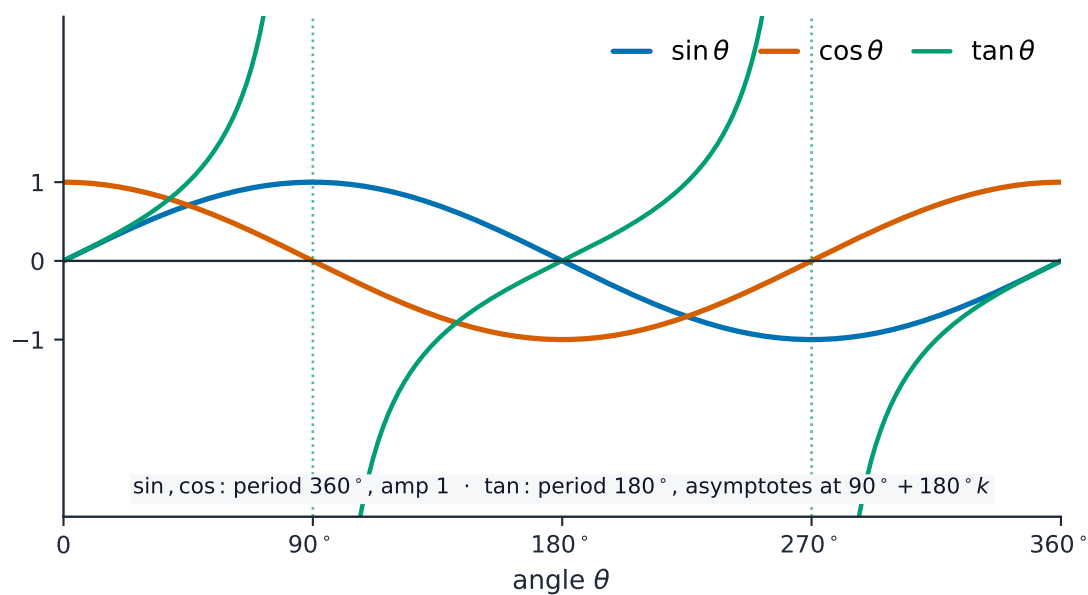
- use $\sec^2 \theta \equiv 1 + \tan^2 \theta$ and $\csc^2 \theta \equiv 1 + \cot^2 \theta$
- apply compound- and double-angle formulae
- write $a \sin \theta + b \cos \theta$ as $R \sin(\theta + \alpha)$ and solve

1 Worked examples

■ Reciprocal identity & R-formula



$\sec \theta = 1/\cos \theta$ rises to an asymptote wherever $\cos \theta = 0$



The underlying sine and cosine graphs

Solve $2 \tan^2 \theta + 3 \sec \theta = 18$ **for** $-180^\circ < \theta < 180^\circ$. Use $\tan^2 \theta = \sec^2 \theta - 1$:
 $2 \sec^2 \theta + 3 \sec \theta - 20 = 0 \Rightarrow (2 \sec \theta - 5)(\sec \theta + 4) = 0$. $\cos \theta = \frac{2}{5}$ or $-\frac{1}{4} \Rightarrow \theta = \pm 66.4^\circ, \pm 104.5^\circ$.

R-formula: $a \sin \theta + b \cos \theta = R \sin(\theta + \alpha)$, $R = \sqrt{a^2 + b^2}$, $\tan \alpha = \frac{b}{a}$.

2 Practice

2.1 Write $\sec^2 \theta$ in terms of $\tan \theta$.

[1]

2.2 Use a double-angle formula to write $\sin 2\theta$ in terms of $\sin \theta$ and $\cos \theta$.

[1]

3 Exam-style questions

3.1 Expressed as $R \sin(\theta + \alpha)$, the value of R for $3 \sin \theta + 4 \cos \theta$ is: [1]

- A 5
 - B 7
 - C $\sqrt{7}$
 - D 12
-

3.2 Express $5 \sin \theta - 12 \cos \theta$ in the form $R \sin(\theta - \alpha)$ with $R > 0$ and $0^\circ < \alpha < 90^\circ$. [3]

3.3 (a) Show that $\frac{1}{1 - \sin \theta} + \frac{1}{1 + \sin \theta} \equiv 2 \sec^2 \theta$. [3]

(b) Hence solve $\frac{1}{1 - \sin \theta} + \frac{1}{1 + \sin \theta} = 8$ for $0^\circ \leq \theta \leq 180^\circ$. [3]

Solutions

2.1 $\sec^2 \theta = 1 + \tan^2 \theta.$

2.2 $\sin 2\theta = 2 \sin \theta \cos \theta.$

3.1 A. $R = \sqrt{3^2 + 4^2} = 5.$

3.2 $R = \sqrt{5^2 + 12^2} = 13; \tan \alpha = \frac{12}{5} \Rightarrow \alpha = 67.4^\circ; 13 \sin(\theta - 67.4^\circ).$

3.3 (a) common denominator: $\frac{(1 + \sin \theta) + (1 - \sin \theta)}{(1 - \sin \theta)(1 + \sin \theta)} = \frac{2}{1 - \sin^2 \theta} = \frac{2}{\cos^2 \theta} = 2 \sec^2 \theta.$

(b) $2 \sec^2 \theta = 8 \Rightarrow \sec^2 \theta = 4 \Rightarrow \cos^2 \theta = \frac{1}{4} \Rightarrow \cos \theta = \pm \frac{1}{2}; \theta = 60^\circ \text{ or } 120^\circ.$