

1.7 Differentiation

Name: _____ Class: _____ Date: _____

Total: 15 marks

KEY WORDS stationary point 驻点 • normal 法线 • chain rule 链式法则 • tangents and normals 切线与法线
• gradient 斜率

Objective

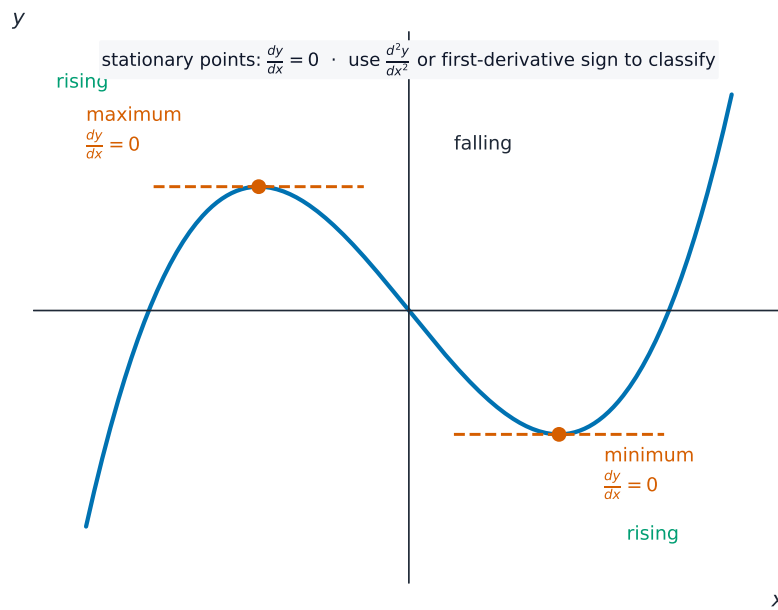
Build the skills to answer exam questions on **differentiation** 微分 — the power and **chain rule** 链式法则, **tangents and normals** 切线与法线, increasing/decreasing, and **stationary points** 驻点.

You must be able to:

- differentiate powers and use the chain rule
- find the equations of a tangent and a normal
- find and classify stationary points (max/min) with $f''(x)$

1 Worked examples

■ Stationary points



At a stationary point $\frac{dy}{dx} = 0$; $f''(x) > 0 \rightarrow \text{minimum}$, $f''(x) < 0 \rightarrow \text{maximum}$

$y = 4x^{1/2} - x$ has a maximum at $x = a$. Find a .

$$\frac{dy}{dx} = 2x^{-1/2} - 1 = 0 \Rightarrow \sqrt{x} = 2 \Rightarrow x = 4, \text{ so } a = 4.$$

Chain rule: $\frac{d}{dx}(2x - 1)^4 = 4(2x - 1)^3 \cdot 2 = 8(2x - 1)^3$.

2 Practice

2.1 Differentiate $y = 3x^4 - \frac{2}{x}$. [2]

2.2 Find $\frac{dy}{dx}$ when $y = (3x + 1)^5$. [2]

3 Exam-style questions

3.1 The gradient of $y = x^3 - 2x$ at $x = 2$ is: [1]

- A 4
 - B 6
 - C 10
 - D 12
-

3.2 A curve has equation $y = x^3 - 6x^2 + 9x + 1$.

(a) Find the coordinates of the two stationary points. [4]

(b) Determine the nature of each stationary point. [2]

3.3 The curve $y = \frac{8}{x}$ passes through $P(2, 4)$. Find the equation of the **normal** to the curve at P . [4]

Solutions

2.1 $\frac{dy}{dx} = 12x^3 + \frac{2}{x^2}.$

2.2 $\frac{dy}{dx} = 5(3x + 1)^4 \cdot 3 = 15(3x + 1)^4.$

3.1 C. $\frac{dy}{dx} = 3x^2 - 2$; at $x = 2$, $= 12 - 2 = 10.$

3.2 (a) $\frac{dy}{dx} = 3x^2 - 12x + 9 = 3(x - 1)(x - 3) = 0$; $x = 1, 3$; $y(1) = 5$, $y(3) = 1$, so $(1, 5)$ and $(3, 1)$.

(b) $\frac{d^2y}{dx^2} = 6x - 12$; at $x = 1$, $= -6 < 0 \rightarrow$ maximum; at $x = 3$, $= 6 > 0 \rightarrow$ minimum.

3.3 $\frac{dy}{dx} = -\frac{8}{x^2}$; at $x = 2$, gradient $= -2$; normal gradient $= \frac{1}{2}$; $y - 4 = \frac{1}{2}(x - 2) \Rightarrow y = \frac{1}{2}x + 3.$