

# 1.4 Circular measure

Name: \_\_\_\_\_ Class: \_\_\_\_\_ Date: \_\_\_\_\_

Total: 13 marks

**KEY WORDS** radian 弧度 • sector 扇形 • arc length 弧长 • sector area 扇形面积 • segment 弓形

## Objective

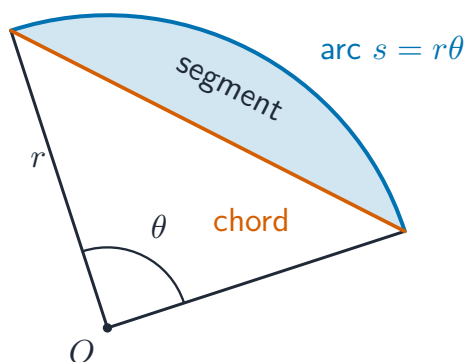
Build the skills to answer exam questions on **circular measure** 弧度制 — **radians** 弧度, **arc length** 弧长, **sector area** 扇形面积 and the area of a **segment** 弓形.

**You must be able to:**

- convert between degrees and radians
- use  $s = r\theta$  and  $A = \frac{1}{2}r^2\theta$  (with  $\theta$  in radians)
- find a segment area as sector minus triangle

## 1 Worked examples

### ■ Sector and segment



*The shaded segment is the part of the sector between the chord and the arc*

For a sector (angle  $\theta$  in radians):  $s = r\theta$ ,  $A = \frac{1}{2}r^2\theta$ . Segment =  $\frac{1}{2}r^2\theta - \frac{1}{2}r^2 \sin \theta = \frac{1}{2}r^2(\theta - \sin \theta)$ .

**A sector has**  $r = 6$ ,  $\theta = \frac{2}{3}\pi$ . Arc =  $6 \cdot \frac{2}{3}\pi = 4\pi$ ; area =  $\frac{1}{2}(36)\frac{2}{3}\pi = 12\pi$ .

## 2 Practice

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**2.1** Convert  $120^\circ$  to radians (exact). [1]

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**2.2** A sector has  $r = 5$  cm and  $\theta = 0.8$  rad. Find its arc length. [1]

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## 3 Exam-style questions

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**3.1** The area of a sector with  $r = 4$  and  $\theta = \frac{\pi}{6}$  is: [1]

- A  $\frac{2\pi}{3}$
  - B  $\frac{4\pi}{3}$
  - C  $\frac{\pi}{3}$
  - D  $2\pi$
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**3.2** A sector  $OAB$  has centre  $O$ , radius 10 cm and angle  $AOB = 1.2$  radians.

(a) Find the perimeter of the sector. [3]

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(b) Find the area of the **segment** between the chord  $AB$  and the arc. [3]

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**3.3** A sector has perimeter 20 cm and area  $16 \text{ cm}^2$ . Show that the radius  $r$  satisfies  $r^2 - 10r + 16 = 0$ , and hence find the two possible radii. [4]

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## Solutions

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**2.1**  $120 \times \frac{\pi}{180} = \frac{2\pi}{3}$ .

**2.2**  $s = r\theta = 5 \times 0.8 = 4$  cm.

**3.1 B.**  $A = \frac{1}{2}(16)\frac{\pi}{6} = \frac{4\pi}{3}$ .

**3.2** (a) arc =  $10 \times 1.2 = 12$ ; perimeter =  $12 + 10 + 10 = 32$  cm.

(b) segment =  $\frac{1}{2}(10^2)(1.2 - \sin 1.2) = 50(1.2 - 0.9320) = 50(0.2680) = 13.4$  cm<sup>2</sup>.

**3.3** perimeter:  $2r + r\theta = 20$ ; area:  $\frac{1}{2}r^2\theta = 16 \Rightarrow r\theta = \frac{32}{r}$ . Substitute:  $2r + \frac{32}{r} = 20 \Rightarrow 2r^2 - 20r + 32 = 0 \Rightarrow r^2 - 10r + 16 = 0$ . Factorise  $(r - 2)(r - 8) = 0 \Rightarrow r = 2$  or  $r = 8$ .