A-LEVEL MATHEMATICS • EXERCISE SHEET

1.3 Coordinate geometry

Name: ______________ Class: __________ Date: ____________ **Total: 16 marks**

KEY WORDS circle 圆 • tangent 切线 • straight line 直线 • gradient 斜率 • perpendicular 垂直

Objective

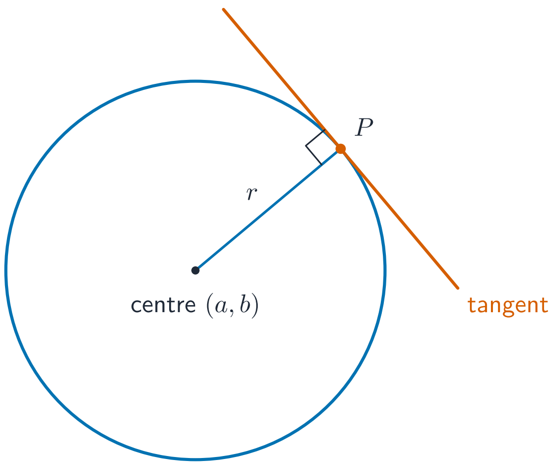
Build the skills to answer exam questions on **coordinate geometry** 坐标几何 — gradients, the equation of a **straight line** 直线, parallel/perpendicular lines, and the equation of a **circle** 圆.

You must be able to:

- find the gradient, midpoint and equation of a line
- use the perpendicular-gradient rule ($m_1 m_2 = -1$)
- find a circle's centre and radius; use the tangent–radius right angle

1 Worked examples

■ Circle and tangent



A tangent touches the circle once and meets the radius at a right angle

$P(1,1)$ and $Q(7,11)$ are ends of a diameter. Find the circle's equation.

Centre = midpoint = $(4,6)$; radius = $\frac{1}{2}\sqrt{6^2+10^2} = \frac{1}{2}\sqrt{136} = \sqrt{34}$. So $(x-4)^2 + (y-6)^2 = 34$.

Perpendicular: a line of gradient 2 has perpendicular gradient $-\frac{1}{2}$.

2 Practice

2.1 Find the gradient of the line joining $(2, -1)$ and $(6, 7)$. [2]

2.2 Write the centre and radius of $(x-3)^2 + (y+2)^2 = 25$. [2]

3 Exam-style questions

3.1 The line through $(0, 4)$ perpendicular to $y = \frac{1}{3}x + 1$ has gradient: [1]

- **A** $\frac{1}{3}$
- **B** 3
- **C** -3
- **D** $-\frac{1}{3}$

3.2 A circle has equation $x^2 + y^2 - 6x + 10y - 2 = 0$.

(a) Find its centre and radius. [3]

(b) Show that the point $(3, 1)$ lies on the circle. [2]

3.3 The points $A(1, 2)$ and $B(5, 10)$ are given.

(a) Find the equation of the perpendicular bisector of AB. [4]

(b) The perpendicular bisector meets the x-axis at C. Find the coordinates of C. [2]

Solutions

2.1 $m = \dfrac{7-(-1)}{6-2} = \dfrac{8}{4} = 2$.

2.2 centre $(3, -2)$; radius 5.

3.1 **C**. Perpendicular gradient $= -1 \div \frac{1}{3} = -3$.

3.2 (a) complete the square: $(x-3)^2 - 9 + (y+5)^2 - 25 - 2 = 0 \Rightarrow (x-3)^2 + (y+5)^2 = 36$; centre $(3, -5)$, radius 6.
(b) substitute $(3, 1)$: $(3-3)^2 + (1+5)^2 = 0 + 36 = 36 = 6^2$; this equals r^2, so $(3, 1)$ lies on the circle.

3.3 (a) midpoint $(3, 6)$; gradient $AB = \dfrac{10-2}{5-1} = 2$, so bisector gradient $= -\frac{1}{2}$; $y - 6 = -\frac{1}{2}(x-3) \Rightarrow y = -\frac{1}{2}x + \frac{15}{2}$.
(b) set $y = 0$: $0 = -\frac{1}{2}x + \frac{15}{2} \Rightarrow x = 15$, so $C(15, 0)$.