

1.2 Functions

Name: _____ Class: _____ Date: _____

Total: 12 marks**KEY WORDS** functions 函数 • transformations 变换 • domain 定义域 • composition 复合 • inverse function 反函数

Objective

Build the skills to answer exam questions on **functions** 函数 — domain and range, **composition** 复合, **inverse functions** 反函数, and **transformations** 变换 of graphs.

You must be able to:

- find the range of a function and a composite fg
- find an inverse f^{-1} and state its domain
- describe and apply graph transformations

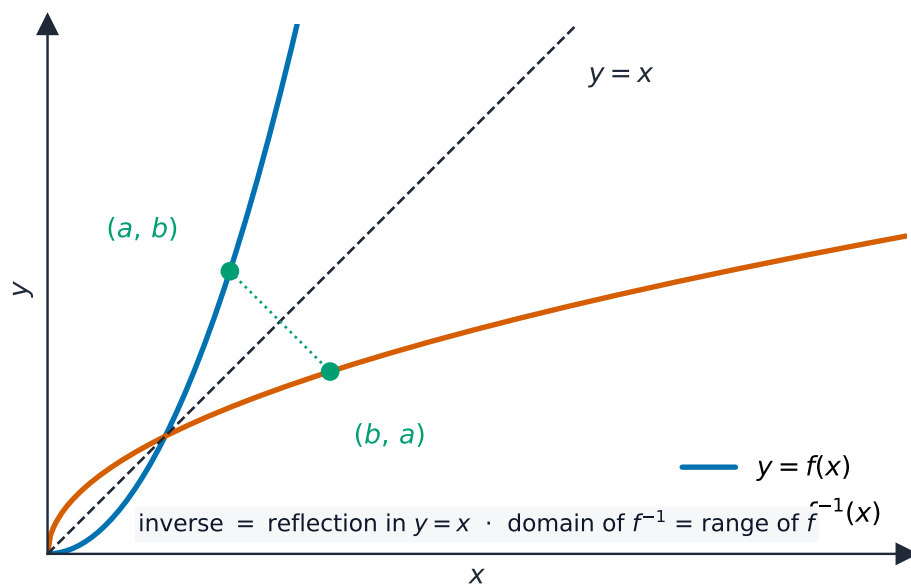
1 Worked examples

■ Inverse & composition

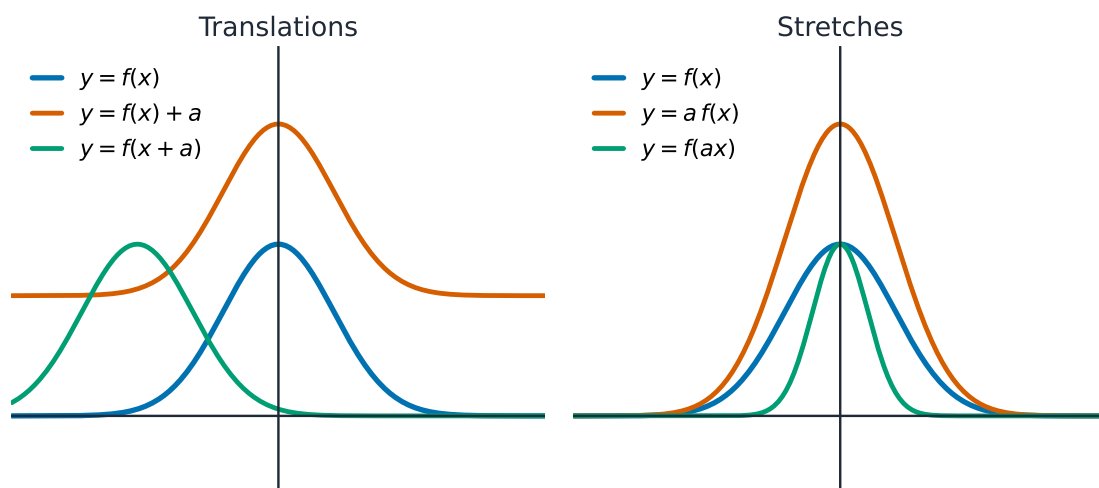
$f(x) = (x + 3)^2 - 12$ for $x \geq 0$. Find $f^{-1}(x)$.

$$y = (x + 3)^2 - 12 \Rightarrow (x + 3)^2 = y + 12 \Rightarrow x = \sqrt{y + 12} - 3.$$

So $f^{-1}(x) = \sqrt{x + 12} - 3$ (positive root, since $x \geq 0$).



The graph of $y = f^{-1}(x)$ is the reflection of $y = f(x)$ in the line $y = x$



$f(x) + a$ moves up · $f(x + a)$ moves left · $af(x)$ stretch y · $f(ax)$ compress x if $a > 1$

$y = f(x) + a$ slides it up; $y = f(x + a)$ slides it left; $y = af(x)$ stretches it in the y -direction

2 Practice

2.1 $f(x) = 2x - 5$ and $g(x) = x^2$. Find $fg(3)$. [2]

2.2 Describe the single transformation mapping $y = f(x)$ to $y = f(x) - 4$. [1]

3 Exam-style questions

3.1 $f(x) = 3 - 2x$. Then $f^{-1}(x)$ is: [1]

- **A** $\frac{3-x}{2}$
 - **B** $\frac{x-3}{2}$
 - **C** $2x-3$
 - **D** $\frac{3+x}{2}$
-

3.2 The function f is defined by $f(x) = x^2 - 6x + 5$ for $x \geq 3$.

(a) Express $f(x)$ in completed-square form and state the range of f . [3]

(b) Find $f^{-1}(x)$ and state its domain. [3]

3.3 The graph of $y = f(x)$ is transformed to $y = 2f(x+1)$. Describe the **two** transformations, in the correct order. [2]

Solutions

2.1 $g(3) = 9$; $f(9) = 2(9) - 5 = 13$.

2.2 a translation **down** by 4.

3.1 A. $y = 3 - 2x \Rightarrow x = \frac{3 - y}{2}$, so $f^{-1}(x) = \frac{3 - x}{2}$.

3.2 (a) $x^2 - 6x + 5 = (x - 3)^2 - 4$; for $x \geq 3$, range is $f(x) \geq -4$.

(b) $y = (x - 3)^2 - 4 \Rightarrow x = 3 + \sqrt{y + 4} \rightarrow f^{-1}(x) = 3 + \sqrt{x + 4}$; domain $x \geq -4$.

3.3 translation left by 1 ($x \rightarrow x + 1$); then a stretch in the y -direction, scale factor 2.