Past-paper walk-through

Linear combinations of random variables ▪ 6.2

eXercise

Questions in this set

- 9709/61 N25 Q1 ▪ 9 marks
- 9709/61 N25 Q6 ▪ 10 marks
- 9709/62 N25 Q2 ▪ 6 marks
- 9709/61 J25 Q4 ▪ 6 marks

Reading the mark scheme

- **B1** a mark that stands on its own – the point is either made or not.
- **M1** a *method* mark: the right approach, even if the number is wrong.
- **A1** an *answer* mark, and it usually needs its M mark first.
- **C1** a working mark on the way to the answer.
- // separates answers the examiner accepts equally.
- **ecf** = error carried forward: a later part is marked on your own earlier answer.
- **owtte** = "or words to that effect" – the wording need not match.

Question 1

9709/61 N25 ▪ Q1 ▪ 9 marks

The random variables X and Y have independent distributions $X \sim \mathrm{Po}(3)$ and $Y \sim \mathrm{Po}(2)$ respectively.

(a) Find $P(2 < X < 5)$. **[2]**

(b) Find $P(X + Y > 2)$. **[3]**

(c) The total of 100 random values of X and 150 random values of Y is denoted by T. Use a suitable approximating distribution to find $P(T < 560)$. **[4]**

Solution 1

(a)

Mark scheme

- $P(2 < X < 5) = P(X = 3) + P(X = 4) = e^{-3}\left(\frac{27}{6} + \frac{81}{24}\right) = 0.392.$

Solution 1

(b) Worked steps

Independent Poisson variables **add**, and their means add with them:

$$X \sim \mathrm{Po}(\lambda_1),\ \ Y \sim \mathrm{Po}(\lambda_2) \ \Rightarrow\ X + Y \sim \mathrm{Po}(\lambda_1 + \lambda_2)$$

Here $X + Y \sim \mathrm{Po}(5)$.

- $P(X + Y > 2) = 1 - P(X + Y \leq 2)$
- $= 1 - e^{-5}\left(1 + 5 + \frac{5^2}{2}\right) = 1 - e^{-5}(18.5) = \mathbf{0.875}$

Solution 1

Two habits. **Use the complement** — "more than 2" is three terms subtracted from 1, where the direct route is an infinite sum. And read the inequality carefully: > 2 excludes 2, so $P(X \leq 2)$ is the right complement; ≥ 2 would need $P(X \leq 1)$. The additive property is only true for **independent** variables, and it is worth saying so: this is the one distribution where the sum stays in the same family.

Mark scheme

- $X + Y \sim \mathrm{Po}(5)$: $P(X + Y > 2) = 1 - e^{-5}(1 + 5 + 12.5) = 0.875.$

Solution 1

(c) Worked steps

A Poisson with a large mean is approximately **normal**, with the variance equal to the mean.

- $E(T) = 100(3) + 150(2) = 600$
- $\mathrm{Var}(T) = 600$ — for a Poisson the variance **equals** the mean, and both add.
- $T \approx N(600, 600)$

Now the probability, with a **continuity correction**:

Solution 1

- $P(T < 560) \approx P\left(Z < \dfrac{559.5 - 600}{\sqrt{600}}\right) = P(Z < -1.653) = \mathbf{0.0492}$

Three marks-worth of detail:

- **State the approximating distribution** with both parameters — the question asks for a suitable one, so naming $N(600, 600)$ is part of the answer.
- **Continuity correction**: $T < 560$ on a discrete variable means $T \leq 559$, so the boundary is **559.5**. Using 560 is the standard error and shifts the answer.
- **Divide by the standard deviation,** $\sqrt{600}$, not by the variance.

Solution 1

The approximation is justified because $\lambda = 600$ is large; a rule of thumb is $\lambda > 15$.

Mark scheme

- $E(T) = 100(3) + 150(2) = 600$, $\mathrm{Var}(T) = 600$, so $T \approx N(600, 600)$. $P(T < 560) \approx P\left(Z < \frac{559.5-600}{\sqrt{600}}\right) = P(Z < -1.653) = 0.0492$.

Question 6

9709/61 N25 ▪ Q6 ▪ 10 marks

The masses, in kilograms, of large and small bags of potatoes have the independent distributions $N(2.5, 0.05)$ and $N(0.8, 0.02)$ respectively.

(a) Find the probability that the total mass of a randomly chosen large bag and a randomly chosen small bag is more than 3.55 kg. **[5]**

(b) Find the probability that the mass of a randomly chosen large bag is less than 3 times the mass of a randomly chosen small bag. **[5]**

Solution 6

(a)

Mark scheme

- $L + S \sim N(3.3, 0.07)$.
  $P(L + S > 3.55) = P\left(Z > \frac{3.55 - 3.3}{\sqrt{0.07}}\right) = P(Z > 0.945) = 0.172.$

Solution 6

(b)

Mark scheme

- $L - 3S \sim N(2.5 - 2.4,\ 0.05 + 9(0.02)) = N(0.1, 0.23)$.
  $P(L < 3S) = P(L - 3S < 0) = P\left(Z < \frac{-0.1}{\sqrt{0.23}}\right) = P(Z < -0.209) = 0.417$.

Question 2

9709/62 N25 ▪ Q2 ▪ 6 marks

The random variable X has a normal distribution with mean 10 and standard deviation 3. The independent random variable Y has a Poisson distribution with mean 4.

(a) Find the standard deviation of $X + Y$. **[3]**

(b) Find the standard deviation of $5X - Y$. **[3]**

Solution 2

(a)

Mark scheme

- $\mathrm{Var}(X + Y) = 3^2 + 4 = 13$, so standard deviation $= \sqrt{13} = 3.61$.

Solution 2

(b)

Mark scheme

- $\mathrm{Var}(5X - Y) = 5^2(3^2) + 4 = 229$, so standard deviation $= \sqrt{229} = 15.1$.

Question 4

9709/61 J25 ▪ Q4 ▪ 6 marks

At an entertainment centre, the cost for using a particular video game is \$0.40 per minute. The number of minutes for which people use the video game has mean 15 and variance 9.

(a) Find the mean and variance of the amount people pay for using the video game. **[3]**

(b) Each day, 35 people independently use the video game. Find the mean and variance of the total amount paid by 35 people. **[3]**

Solution 4

(a)

Mark scheme

- Mean $= 0.40 \times 15 = 6$; variance $= 0.40^2 \times 9 = 1.44$.

Solution 4

(b)

Mark scheme

- Mean $= 35 \times 6 = 210$; variance $= 35 \times 1.44 = 50.4$.