Past-paper walk-through

Kinematics of motion in a straight line ▪ 4.2

eXercise

Questions in this set

- 9709/41 N25 Q7 ▪ 9 marks
- 9709/42 N25 Q1 ▪ 3 marks
- 9709/42 N25 Q3 ▪ 8 marks
- 9709/42 N25 Q4 ▪ 9 marks
- 9709/42 N25 Q6 ▪ 8 marks
- 9709/41 J25 Q3 ▪ 7 marks
- 9709/41 J25 Q7 ▪ 13 marks

Reading the mark scheme

- **B1** a mark that stands on its own – the point is either made or not.
- **M1** a *method* mark: the right approach, even if the number is wrong.
- **A1** an *answer* mark, and it usually needs its M mark first.
- **C1** a working mark on the way to the answer.
- // separates answers the examiner accepts equally.
- **ecf** = error carried forward: a later part is marked on your own earlier answer.
- **owtte** = “or words to that effect” – the wording need not match.

Question 7

9709/41 N25 ▪ Q7 ▪ 9 marks

A particle P starts from a point O and moves in a straight line. The velocity v m s^{-1} of P, at time t s after leaving O, is given by $v = \frac{1}{3}(2t-3)(t-4)$.

(a) Find the acceleration of P when $t = 2$. **[2]**

(b) Find the total distance travelled by P in the first 3 seconds of its motion. **[5]**

(c) Determine whether P returns to O. **[2]**

Solution 7

(a)

Mark scheme

- $a = \dfrac{dv}{dt} = \frac{1}{3}(4t - 11)$; at $t = 2$, $a = -1$ m s^{-2}.

Solution 7

(b) Worked steps

Total distance is not displacement. The velocity changes sign, so the particle reverses, and the two legs must be measured separately and **added as positive amounts**.

1. **Find where it turns.** $v = \frac{1}{3}(2t-3)(t-4) = 0$ at $t = 1.5$ and $t = 4$. Only $t = 1.5$ lies in the first 3 seconds.
2. **Integrate v to get displacement**, with $s = 0$ at $t = 0$.
3. **Evaluate at the turning point and at the end**: $s(1.5) = 2.625$ and $s(3) = 1.5$.
4. **Add the legs as distances**:
    - out: $2.625 - 0 = 2.625$ m
    - back: $|1.5 - 2.625| = 1.125$ m
    - total $= 2.625 + 1.125 = \mathbf{3.75}$ m

Solution 7

The mark that separates answers here is step 1. Evaluating $s(3)$ alone gives 1.5 m — the **displacement**, which is what the particle's net movement is, not how far it travelled. The word "total distance" is the instruction to look for a sign change first. Take the modulus of the second leg. Adding $1.5 - 2.625$ as a signed quantity just recovers the displacement again.

Mark scheme

- $v = 0$ at $t = 1.5$. $s(1.5) = 2.625$, $s(3) = 1.5$. Total distance $= 2.625 + (2.625 - 1.5) = 3.75$ m.

Solution 7

(c) Worked steps

"Does it return to O" means: **is there a positive t with $s(t) = 0$?**

- Set the displacement to zero and clear the fractions: $4t^2 - 33t + 72 = 0$ (having divided out the factor of t, which is the start at $t = 0$).
- **Test the discriminant**: $b^2 - 4ac = 33^2 - 4(4)(72) = 1089 - 1152 = -63 < 0$.
- No real roots, so no positive time gives $s = 0$: P **does not return to** O.

The discriminant is the whole technique — it answers "are there solutions?" without finding them, which is exactly the question. Solving numerically and reporting "no answer on my calculator" is not a proof.

Solution 7

Divide out the $t = 0$ root first. That root is the particle *starting* at O, and leaving it in makes the equation appear to have a solution when it has no *useful* one.

Mark scheme

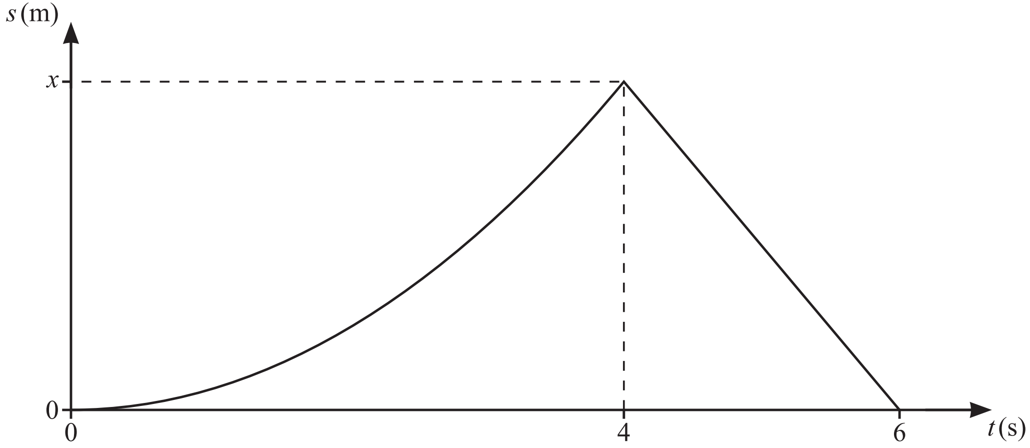
- $s(t) = 0 \Rightarrow 4t^2 - 33t + 72 = 0$ (for $t \neq 0$); discriminant $= 1089 - 1152 < 0$, so no $t > 0$ gives $s = 0$ — P does not return to O.

Question 1

9709/42 N25 ▪ Q1 ▪ 3 marks

The diagram shows the displacement-time graph for the motion of a particle. The particle starts from rest at a point O and travels with constant acceleration a m s^{-2}, taking 4 s to move a distance of x m. The particle then returns to O with constant speed of 40 m s^{-1}, over a period of 2 s. Find the value of x and the value of a. **[3]**

Question 1



Solution 1

Mark scheme

- Return leg: $x = 40 \times 2 = 80$ m. Acceleration leg from rest: $80 = \frac{1}{2}a(4^2) \Rightarrow a = 10$. So $x = 80$, $a = 10$ m s^{-2}.

Question 3

9709/42 N25 ▪ Q3 ▪ 8 marks

A car of mass 1400 kg travelling on a straight horizontal road accelerates uniformly from a speed of 15 m s^{-1} to 20 m s^{-1} over a distance of 175 m.

(a) Find the time taken to travel the 175 m. **[2]**

(b)(i) Use an energy method throughout to find the average power of the car's engine as the speed increases from 15 m s^{-1} to 20 m s^{-1}. **[4]**

(b)(ii) Find the steady speed that the car could maintain on the horizontal road if the engine is working at the power found in (b)(i). **[2]**

Solution 3

(a) Worked steps

With **uniform** acceleration the average speed is the mean of the initial and final speeds — no need to find a at all.

- $\bar{v} = \dfrac{15 + 20}{2} = 17.5\ \mathrm{m\,s^{-1}}$
- $t = \dfrac{175}{17.5} = \mathbf{10}\ \mathrm{s}$

Solution 3

That shortcut works **only** for constant acceleration, which is why the question says "uniformly". The alternative is $v^2 = u^2 + 2as$ to find a, then $v = u + at$ — three lines instead of two, and the same answer.

Mark scheme

- Average speed $= \frac{15+20}{2} = 17.5$, so time $= 175/17.5 = 10$ s.

Solution 3

(b)(i) Worked steps

The question says **use an energy method throughout**, so no forces and no $F = ma$ — account for the energy instead.

$$\text{work by engine} = \text{gain in kinetic energy} + \text{work against resistance}$$

- Gain in KE: $\frac{1}{2}(1400)\left(20^2 - 15^2\right) = 700(400 - 225) = 122\,500$ J
- Work against resistance: $800 \times 175 = 140\,000$ J
- Engine work: $122\,500 + 140\,000 = 262\,500$ J
- Average power: $\dfrac{262\,500}{10} = \mathbf{26\,250}$ W

Solution 3

Two things carry the marks. **Both terms** — the resistance does not vanish because the car is speeding up, and it is the larger of the two here. And **use the time from part (a)**: power is work per unit time, so the 10 s is what turns joules into watts. $\frac{1}{2}m\left(v^2 - u^2\right)$ is one subtraction of squares, not two separate energies subtracted after rounding.

Mark scheme

- Work-energy: engine work $= \Delta KE +$ work against resistance $= \frac{1}{2}(1400)(20^2 - 15^2) + 800(175) = 122500 + 140000 = 262500$ J. Average power $= 262500/10 = 26250$ W.

Solution 3

(b)(ii) Worked steps

At a **steady** speed there is no acceleration, so the driving force exactly balances the resistance.

- $F = 800$ N
- $P = Fv$, so $26\,250 = 800v$
- $v = \mathbf{32.8}$ m s^{-1}

Solution 3

The reasoning worth writing down is the first line: constant speed means zero net force, which is what lets you replace the unknown driving force with the known resistance. Without it the equation has two unknowns.
Sense-check: 32.8 m s^{-1} is about 118 km/h, a plausible top speed for a car at that power.

Mark scheme

- At a steady speed, driving force $=$ resistance $= 800$ N, so $26250 = 800v \Rightarrow v = 32.8$ m s^{-1}.

Question 4

9709/42 N25 ▪ Q4 ▪ 9 marks

A particle P of mass 0.1 kg is projected vertically upwards with speed 30 m s^{-1} from horizontal ground. At the same instant a particle Q of mass 0.4 kg is projected vertically upwards with speed 10 m s^{-1} from a height of 15 m above the ground. P and Q move in the same vertical line.

(a) Find the height above the ground at which P and Q collide. **[4]**

(b) When P and Q collide, they coalesce. Find the speed of the combined particle at the instant that it reaches the ground. **[5]**

Solution 4

(a) Worked steps

Write a **displacement equation for each particle from the ground**, then set them equal.

- P from the ground: $s_P = 30t - 5t^2$
- Q from 15 m up: $s_Q = 15 + 10t - 5t^2$

They collide when $s_P = s_Q$:

- $30t - 5t^2 = 15 + 10t - 5t^2$
- The t^2 terms **cancel** — both particles are in the same gravitational field — leaving $20t = 15$, so $t = 0.75$ s.
- Height: $s_P = 30(0.75) - 5(0.75)^2 = 22.5 - 2.8125 = \mathbf{19.7}$ m

Solution 4

Measuring **both** displacements from the same origin is what makes this a one-line cancellation. Taking Q's displacement from its own starting point leaves the 15 m out and gives the wrong time.

Mark scheme

- $30t - 5t^2 = 15 + 10t - 5t^2 \Rightarrow t = 0.75$ s. Height $= 30(0.75) - 5(0.75)^2 = 19.7$ m.

Solution 4

(b) Worked steps

Three stages, and each needs its own principle.

1. Velocities just before impact — use $v = u - gt$ for each:

- $v_P = 30 - 10(0.75) = 22.5\ \mathrm{m\,s^{-1}}$ (upward)
- $v_Q = 10 - 10(0.75) = 2.5\ \mathrm{m\,s^{-1}}$ (upward)

2. Conservation of momentum at the coalescence — they stick together, so one combined mass:

- $0.1(22.5) + 0.4(2.5) = (0.1 + 0.4)v$
- $2.25 + 1.0 = 0.5v$, so $v = 6.5\ \mathrm{m\,s^{-1}}$ upward.

Solution 4

3. The fall to the ground — $v^2 = u^2 + 2as$, taking downward as positive after the combined particle has risen and returned:

- $v^2 = 6.5^2 + 2(10)(19.6875) = 42.25 + 393.75 = 436$
- $v = \mathbf{20.9}\ \mathrm{m\,s^{-1}}$

Solution 4

Two things to be deliberate about. **Both velocities are upward** at the collision, so the momenta **add** — a sign error here is the commonest way to lose the second stage. And the $v^2 = u^2 + 2as$ step needs no separate treatment of the rise and fall: the particle returns to the collision height with the same speed, so starting the fall from there with $u = 6.5$ upward is equivalent and much shorter.

Mark scheme

- At collision $v_P = 22.5$, $v_Q = 2.5$ (both upward). Momentum: $0.1(22.5) + 0.4(2.5) = 0.5v \Rightarrow v = 6.5$ m s^{-1} up. Falling from 19.6875 m: $v^2 = 6.5^2 + 2(10)(19.6875) = 436 \Rightarrow v = 20.9$ m s^{-1}.

Question 6

9709/42 N25 ▪ Q6 ▪ 8 marks

A particle starts from rest at a point O. The acceleration of the particle at time t s after leaving O is a m s^{-2}, where $a = 2(t+1)^{-\frac{1}{2}} - 1$ for $t \geqslant 0$. Find the distance that the particle travels from O until the time at which its acceleration is zero. **[8]**

Solution 6

Worked steps

Three steps, and the constants of integration are where the marks live.

1. **Find the time.** Acceleration is zero when
   - $2(t+1)^{-1/2} = 1$, so $(t+1)^{1/2} = 2$ and $t+1 = 4$: $t = 3$.
2. **Integrate to get velocity**, using $v = 0$ at $t = 0$:
   - $v = \displaystyle\int \left(2(t+1)^{-1/2} - 1\right) dt = 4(t+1)^{1/2} - t + c$
   - At $t = 0$: $0 = 4(1) - 0 + c$, so $c = -4$.
   - $v = 4(t+1)^{1/2} - t - 4$

Solution 6

3. Integrate again to get displacement, using $s = 0$ at $t = 0$:

- $s = \frac{8}{3}(t+1)^{3/2} - \frac{1}{2}t^2 - 4t + k$
- At $t = 0$: $0 = \frac{8}{3} + k$, so $k = -\frac{8}{3}$.
- At $t = 3$: $s = \frac{8}{3}(8) - \frac{1}{2}(9) - 12 - \frac{8}{3} = \frac{64}{3} - 4.5 - 12 - \frac{8}{3} = \mathbf{\frac{13}{6}} \approx 2.17$ m

Three points decide the marks:

- **Evaluate both constants.** "Starts from rest at O" gives you $v(0) = 0$ **and** $s(0) = 0$, and each is worth a mark. Omitting them is the standard error in variable- acceleration questions.
- **Integrate** $(t+1)^{-1/2}$ **as a chain**: the inner function differentiates to 1, so the integral is $2 \times 2(t+1)^{1/2}$.
- **The velocity never changes sign** on $0 \leq t \leq 3$, so the distance equals the displacement here — worth confirming rather than assuming, since a sign change would mean splitting the interval.

Solution 6

Mark scheme

- $a = 0$ when $2(t+1)^{-1/2} = 1 \Rightarrow t = 3$. $v = \int a\,dt = 4(t+1)^{1/2} - t - 4$ (with $v(0) = 0$); $s = \int v\,dt = \frac{8}{3}(t+1)^{3/2} - \frac{1}{2}t^2 - 4t - \frac{8}{3}$ (with $s(0) = 0$). At $t = 3$: $s = \frac{13}{6} \approx 2.17$ m.

Question 3

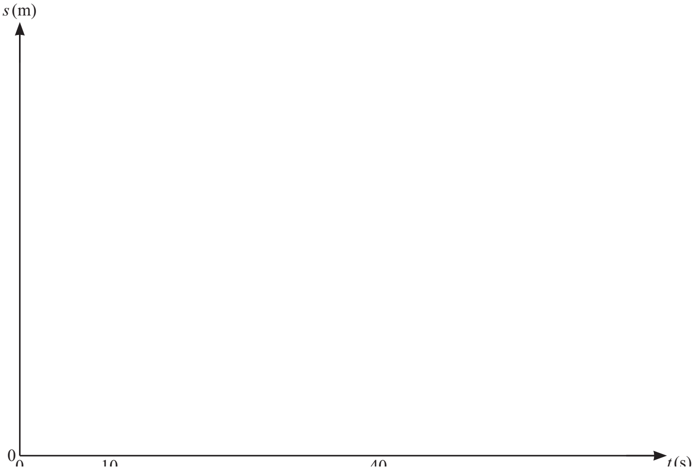
9709/41 J25 ▪ Q3 ▪ 7 marks

The diagram shows the velocity-time graph of the motion of a cyclist. The graph consists of three straight line segments. The cyclist passes a point O with speed 3 m s^{-1} and then accelerates for 10 s with constant acceleration 0.5 m s^{-2}. He then travels at constant speed for 30 s before decelerating, coming to rest at point P, covering a distance of 80 m whilst decelerating.

(a) Find the total time taken for the journey from O to P. **[3]**

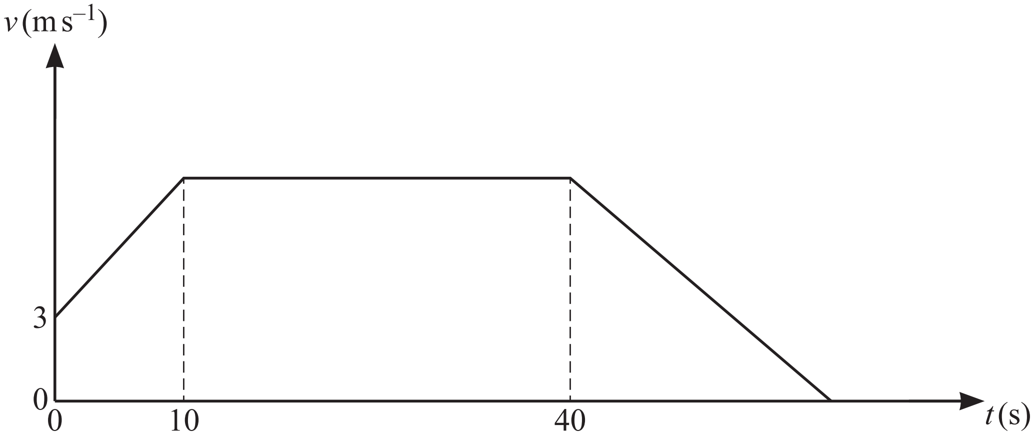
(b) On the given axes, sketch a displacement-time graph for the cyclist's journey from O to P, showing on your graph the distances travelled after 10 s and 40 s. **[4]**

Question 3



Question 3

Question 3



Solution 3

(a) Worked steps

On a **velocity-time** graph, the **area** under the line is the distance. Take the phases in order.

- **Phase 1** — accelerating for 10 s at $0.5\ \mathrm{m\,s^{-2}}$ from $3\ \mathrm{m\,s^{-1}}$: the final speed is $3 + 0.5(10) = 8\ \mathrm{m\,s^{-1}}$.
- **Phase 2** — constant $8\ \mathrm{m\,s^{-1}}$ for 30 s, ending at $t = 40$ s.
- **Phase 3** — decelerating from 8 to rest, covering 80 m. That phase is a triangle: $\frac{1}{2}(8)(T - 40) = 80$, so $T - 40 = 20$.

Solution 3

Total time $= 60$ s.

The step worth writing out is the first: the deceleration phase cannot be started until its initial speed is known, and that comes from phase 1. Working the phases in order is what keeps that visible.

Mark scheme

- Velocity at $t = 10$ is $3 + 0.5(10) = 8$ m s^{-1}. Deceleration phase: $\frac{1}{2}(8)(T - 40) = 80 \Rightarrow T - 40 = 20$. Total time $= 60$ s.

Solution 3

(b) Worked steps

A displacement-time graph is the **integral** of the velocity one, so translate each phase's *velocity* into the *shape* of the displacement curve:

- **Constant acceleration** $\rightarrow$ a **curve of increasing gradient** (a parabola).
- **Constant velocity** $\rightarrow$ a **straight line** whose gradient is that velocity.
- **Constant deceleration to rest** $\rightarrow$ a **curve of decreasing gradient**, flattening to horizontal at the end.

Mark the two distances the question asks for:

- After 10 s: the trapezium $\frac{1}{2}(3+8)(10) = \mathbf{55}$ m
- After 40 s: $55 + 8(30) = \mathbf{295}$ m
- At 60 s: $295 + 80 = 375$ m

Solution 3

Two features are marked and are easy to miss. The curve must be **continuous and smooth at the joins** — displacement cannot jump, and the gradient must match the velocity on both sides. And it **never decreases**: the cyclist always moves forwards, so the graph rises throughout and merely flattens at the end.

Mark scheme

- Distance after 10 s $= \frac{1}{2}(3+8)(10) = 55$ m; after 40 s $= 55 + 8(30) = 295$ m. Graph: quadratic with increasing gradient (0–10 s), straight line (10–40 s), then curve of decreasing gradient to rest.

Question 7

9709/41 J25 ▪ Q7 ▪ 13 marks

A particle X moves along a straight track, starting from a point O at time $t = 0$. The displacement of X from O at time t s is s m, where $s = 3t^{\frac{3}{2}} - 6t$.

(a) Find the time at which X is instantaneously at rest, and hence find the total distance travelled by X between $t = 0$ and $t = 16$. **[6]**

(b) A second particle Y moves along another straight track, starting from a point P at time $t = 0$. The acceleration of Y at time t s is a m s^{-2}, where $a = 0.8 - 0.6t$. The velocity of Y when it leaves P is 7.5 m s^{-1}. When the velocity of Y is -9.6 m s^{-1}, show that the displacement of X from O is equal to the displacement of Y from P. **[7]**

Solution 7

(a) Worked steps

Total distance is not displacement, so find the reversal first.

- $s = 3t^{3/2} - 6t$, so $v = \dfrac{ds}{dt} = \frac{9}{2}t^{1/2} - 6$.
- At rest: $\frac{9}{2}t^{1/2} = 6$, so $t^{1/2} = \frac{4}{3}$ and $t = \dfrac{16}{9}$ s.

Now evaluate the displacement at the turning point and at the end:

- $s\left(\frac{16}{9}\right) = -\dfrac{32}{9}$ — the particle has moved **backwards** first.
- $s(16) = 3(64) - 96 = 96$ m.

Add the two legs **as distances**:

- back: $\left|-\frac{32}{9}\right| = \frac{32}{9}$

Solution 7

- forward: $96 - (-\frac{32}{9}) = \frac{896}{9}$
- total $= \frac{32}{9} + \frac{896}{9} = \dfrac{928}{9} \approx \mathbf{103}$ m

Reporting $s(16) = 96$ alone gives the displacement and misses the mark: the particle went backwards for the first $\frac{16}{9}$ seconds, and that ground was covered twice.

Note $3t^{3/2}$ differentiates to $\frac{9}{2}t^{1/2}$ — multiply by the power and reduce it by one, and $3 \times \frac{3}{2} = \frac{9}{2}$. **Mark scheme**

- $v = \frac{9}{2}t^{1/2} - 6 = 0 \Rightarrow t = \frac{16}{9}$ s. $s(\frac{16}{9}) = -\frac{32}{9}$, $s(16) = 96$. Total distance $= \frac{32}{9} + \frac{896}{9} = \frac{928}{9} \approx 103$ m.

Solution 7

(b) Worked steps

A *show that*, so build both displacements and evaluate them at the same time.

Find the time. Integrate Y's acceleration, using $v = 7.5$ at $t = 0$:

- $v_Y = 0.8t - 0.3t^2 + 7.5$
- Set $v_Y = -9.6$: $0.3t^2 - 0.8t - 17.1 = 0$, so $3t^2 - 8t - 171 = 0$ and $(t-9)(3t+19) = 0$, giving $t = 9$ s (the negative root has no meaning).

Evaluate both displacements at $t = 9$.

- $s_X = 3(9)^{3/2} - 6(9) = 3(27) - 54 = 27$ m
- $s_Y = \int v_Y \, dt = 0.4t^2 - 0.1t^3 + 7.5t$ (with $s_Y(0) = 0$), so $s_Y(9) = 0.4(81) - 0.1(729) + 67.5 = 32.4 - 72.9 + 67.5 = 27$ m

Solution 7

They are equal. **Shown.**
Two things carry it. The **constant of integration** for v_Y comes from the stated launch velocity of $7.5\ \mathrm{m\,s^{-1}}$, and the one for s_Y from starting at P — both are marks. And $9^{3/2} = \left(\sqrt{9}\right)^3 = 27$; taking the square root first is much safer than cubing 9 and then rooting. A *show that* needs both values written out and the conclusion stated. Reaching 27 twice and stopping leaves the marker to draw the inference.

Mark scheme

- $v_Y = 0.8t - 0.3t^2 + 7.5$; $v_Y = -9.6 \Rightarrow 0.3t^2 - 0.8t - 17.1 = 0 \Rightarrow t = 9$. Then $s_X(9) = 3(9^{3/2}) - 6(9) = 27$ m and $s_Y(9) = 0.4(81) - 0.1(729) + 7.5(9) = 27$ m — equal.