Past-paper walk-through

Numerical solution of equations ▪ 2.6

eXercise

Questions in this set

- 9709/21 N25 Q7 ▪ 11 marks
- 9709/22 N25 Q6 ▪ 9 marks
- 9709/21 J25 Q3 ▪ 7 marks
- 9709/22 J25 Q4 ▪ 8 marks

Reading the mark scheme

- **B1** a mark that stands on its own – the point is either made or not.
- **M1** a *method* mark: the right approach, even if the number is wrong.
- **A1** an *answer* mark, and it usually needs its M mark first.
- **C1** a working mark on the way to the answer.
- // separates answers the examiner accepts equally.
- **ecf** = error carried forward: a later part is marked on your own earlier answer.
- **owtte** = "or words to that effect" – the wording need not match.

Question 7

9709/21 N25 ▪ Q7 ▪ 11 marks

The polynomial $p(x)$ is defined by $p(x) = 2x^4 + kx^3 + kx^2 + 17x + 18$, where k is a constant. It is given that $(x + 2)$ is a factor of $p(x)$.

(a) Find the value of k. **[2]**

(b) It is given that the equation $p(x) = 0$ has exactly two real roots, denoted by α and β, where α is an integer and β is not an integer. State the value of α and show that β satisfies the equation $x = \sqrt[3]{-2x - 4.5}$. **[4]**

(c) Show by calculation that $-1.4 < \beta < -1.0$. **[2]**

(d) Use an iterative formula, based on the equation in part (b), to find the value of β correct to 3 significant figures. Give the result of each iteration to 5 significant figures. **[3]**

Solution 7

(a)

Mark scheme

- $p(-2) = 32 - 8k + 4k - 34 + 18 = 16 - 4k = 0 \Rightarrow k = 4.$

Solution 7

(b)

Mark scheme

- $\alpha = -2$. Dividing $p(x)$ by $(x+2)$ leaves $2x^3 + 4x + 9 = 0 \Rightarrow x^3 = -2x - 4.5 \Rightarrow x = \sqrt[3]{-2x - 4.5}$, which β satisfies.

Solution 7

(c)

Mark scheme

- $f(x) = 2x^3 + 4x + 9$: $f(-1.4) = -2.09 < 0$ and $f(-1.0) = 3 > 0$, so $-1.4 < \beta < -1.0$.

Solution 7

(d)

Mark scheme

- Iterating $x_{n+1} = \sqrt[3]{-2x_n - 4.5}$ from $x_0 = -1.2$: -1.2806, -1.2475, -1.2610, $-1.2553, \ldots \Rightarrow \beta = -1.26$ (3 s.f.).

Question 6

9709/22 N25 ▪ Q6 ▪ 9 marks

It is given that $\int_{-2a}^{a} \left(\frac{1}{2}e^{2x} + \frac{1}{4}e^{-x}\right) dx = 5$, where a is a positive constant.

(a) Show that $a = \frac{1}{2}\ln\left(10 + \frac{1}{2}e^{-a} + \frac{1}{2}e^{-4a}\right)$. **[4]**

(b) Hence show by calculation that the value of a lies between 1.0 and 1.2. **[2]**

(c) Use the iterative formula $a_{n+1} = \frac{1}{2}\ln\left(10 + \frac{1}{2}e^{-a_n} + \frac{1}{2}e^{-4a_n}\right)$ to find the value of a correct to 4 significant figures. Give the result of each iteration to 6 significant figures. **[3]**

Solution 6

(a)

Mark scheme

- $\int \left(\frac{1}{2}e^{2x} + \frac{1}{4}e^{-x}\right) dx = \frac{1}{4}e^{2x} - \frac{1}{4}e^{-x}$. Evaluating $-2a$ to a gives $\frac{1}{2}e^{2a} - \frac{1}{4}e^{-a} - \frac{1}{4}e^{-4a} = 5 \Rightarrow e^{2a} = 10 + \frac{1}{2}e^{-a} + \frac{1}{2}e^{-4a} \Rightarrow a = \frac{1}{2}\ln(\cdots)$. AG.

Solution 6

(b)

Mark scheme

- $F(a) = \frac{1}{2}\ln(10 + \frac{1}{2}e^{-a} + \frac{1}{2}e^{-4a}) - a$: $F(1.0) = +0.161 > 0$ and $F(1.2) = -0.041 < 0$; sign change $\Rightarrow$ root between 1.0 and 1.2.

Solution 6

(c)

Mark scheme

- Iterating from $a_0 = 1.1$: $a_1 = 1.15986$, $a_2 = 1.15930$, $a_3 = 1.15936, \ldots \Rightarrow a = 1.159$ (4 s.f.).

Question 3

9709/21 J25 ▪ Q3 ▪ 7 marks

(a) Sketch, on a single diagram, the graphs of $y = 3e^{-2x}$ and $y = \sec x$ for values of x such that $0 \leqslant x < \frac{1}{2}\pi$. **[2]**

(b) Show that the x-coordinate of the point of intersection of the two graphs satisfies the equation $x = \frac{1}{2}\ln(3\cos x)$. **[2]**

(c) Use an iterative formula, based on the equation in part (b), to find the x-coordinate of the point of intersection correct to 3 decimal places. Give the result of each iteration to 5 decimal places. **[3]**

Solution 3

(a)

Mark scheme

- $y = 3e^{-2x}$ is a decreasing positive curve through $(0,3)$; $y = \sec x$ rises from $(0,1)$ to $+\infty$ as $x \to \frac{1}{2}\pi$. They meet once.

Solution 3

(b)

Mark scheme

- At intersection
  $3e^{-2x} = \sec x \Rightarrow e^{2x} = 3\cos x \Rightarrow 2x = \ln(3\cos x) \Rightarrow x = \frac{1}{2}\ln(3\cos x)$ (AG).

Solution 3

(c)

Mark scheme

- Iterating $x_{n+1} = \frac{1}{2}\ln(3\cos x_n)$ (radian mode) converges to $x = 0.487$ (3 d.p.).

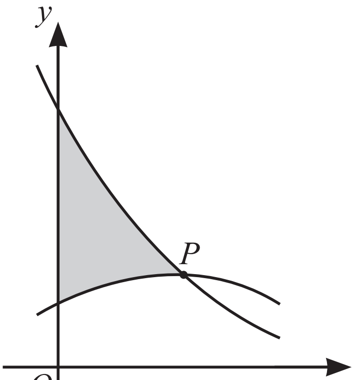
Question 4

9709/22 J25 ▪ Q4 ▪ 8 marks

The diagram shows parts of the curves with equations $y = 4e^{-2x}$ and $y = 1 + 0.5\sin 3x$. Point P is a point of intersection of the curves, and the shaded region is bounded by the two curves and the y-axis.

(a) Show that the x-coordinate of P satisfies the equation $x = -0.5\ln(0.25 + 0.125\sin 3x)$. **[1]**

(b) Use an iterative formula, based on the equation in part (a), to find the x-coordinate of P correct to 4 significant figures. Use an initial value of 0.5 and



Solution 4

(a)

Mark scheme

- At P: $4e^{-2x} = 1 + 0.5\sin 3x \Rightarrow e^{-2x} = 0.25 + 0.125\sin 3x \Rightarrow x = -0.5\ln(0.25 + 0.125\sin 3x)$ (AG).

Solution 4

(b)

Mark scheme

- Iterating $x_{n+1} = -0.5\ln(0.25 + 0.125\sin 3x_n)$ from $x_0 = 0.5$ converges to $x = 0.4912$ (4 s.f.).

Solution 4

(c)

Mark scheme

- Area $= \int_0^{0.4912} \left[4e^{-2x} - (1 + 0.5\sin 3x)\right] dx = \left[-2e^{-2x} - x + \frac{1}{6}\cos 3x\right]_0^{0.4912} = 0.61$ (2 s.f.).