Past-paper walk-through

Logarithmic and exponential functions ▪ 2.2

eXercise

Questions in this set

- 9709/21 N25 Q2 ▪ 3 marks
- 9709/21 N25 Q5 ▪ 7 marks
- 9709/22 N25 Q1 ▪ 4 marks
- 9709/21 J25 Q2 ▪ 6 marks
- 9709/22 J25 Q6 ▪ 9 marks

Reading the mark scheme

- **B1** a mark that stands on its own – the point is either made or not.
- **M1** a *method* mark: the right approach, even if the number is wrong.
- **A1** an *answer* mark, and it usually needs its M mark first.
- **C1** a working mark on the way to the answer.
- // separates answers the examiner accepts equally.
- **ecf** = error carried forward: a later part is marked on your own earlier answer.
- **owtte** = "or words to that effect" – the wording need not match.

Question 2

9709/21 N25 ▪ Q2 ▪ 3 marks

Solve the equation $e^{2x}(e^{2x} - 8) = 48$. **[3]**

Solution 2

Mark scheme

- Let $y = e^{2x}$: $y^2 - 8y - 48 = 0 \Rightarrow (y - 12)(y + 4) = 0 \Rightarrow y = 12$ (reject -4). $e^{2x} = 12 \Rightarrow x = \frac{1}{2}\ln 12$.

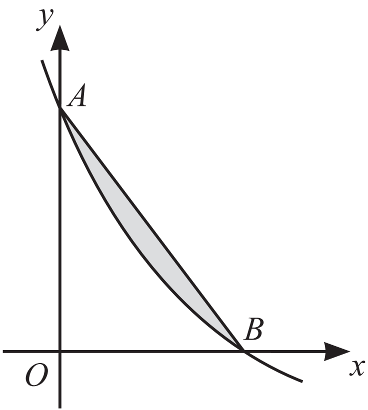
Question 5

9709/21 N25 ▪ Q5 ▪ 7 marks

The diagram shows the curve with equation $y = 8e^{-\frac{1}{2}x} - 1$. The curve meets the axes at the points A and B. The shaded region is bounded by the curve and the line segment AB.

(a) Show that the x-coordinate of B is $6\ln 2$. **[2]**

(b) Find the area of the shaded region. Give your answer in the form $p\ln 2 - q$, where p and q are positive integers. **[5]**



Solution 5

(a)

Mark scheme

- $y = 0 \Rightarrow 8e^{-x/2} = 1 \Rightarrow e^{-x/2} = \frac{1}{8} \Rightarrow -\frac{x}{2} = -3\ln 2 \Rightarrow x = 6\ln 2.$

Solution 5

(b)

Mark scheme

- $A = (0, 7)$, $B = (6\ln 2, 0)$. Triangle area $= \frac{1}{2}(6\ln 2)(7) = 21\ln 2$; $\int_0^{6\ln 2}(8e^{-x/2} - 1)\,dx = 14 - 6\ln 2$. Shaded area $= 21\ln 2 - (14 - 6\ln 2) = 27\ln 2 - 14$.

Question 1

9709/22 N25 ▪ Q1 ▪ 4 marks

Solve the equation $\ln(3x + 5) - \ln(x - 2) = 4$. Give your answer in an exact form. **[4]**

Solution 1

Mark scheme

- $\ln \dfrac{3x+5}{x-2} = 4 \Rightarrow \dfrac{3x+5}{x-2} = e^4 \Rightarrow x = \dfrac{2e^4+5}{e^4-3}$.

Question 2

9709/21 J25 ▪ Q2 ▪ 6 marks

(a) Use logarithms to solve the inequality $4^x < 0.05$. Give your answer in the form $x < a$, where the value of a is correct to 3 significant figures. **[2]**

(b) Solve the inequality $|3x + 8| < 9$. **[3]**

(c) Hence state the integers that satisfy both of the inequalities in parts (a) and (b). **[1]**

Solution 2

(a)

Mark scheme

- $x \ln 4 < \ln 0.05 \Rightarrow x < \dfrac{\ln 0.05}{\ln 4} = -2.16$ (3 s.f.).

Solution 2

(b)

Mark scheme

- $3x + 8 = \pm 9 \Rightarrow x = -\frac{17}{3}$ or $\frac{1}{3}$; so $-\frac{17}{3} < x < \frac{1}{3}$.

Solution 2

(c)

Mark scheme

- Integers satisfying both inequalities: $-5,\ -4,\ -3.$

Question 6

9709/22 J25 ▪ Q6 ▪ 9 marks

A curve has equation $(x^2 - 3)\ln y + 6x = 14$.

(a) Show that there is no point on the curve at which the y-coordinate is e^{-1}. **[3]**

(b) Find the equation of the tangent to the curve at the point $(2, e^2)$. Give your answer in the form $y = mx + c$, where m and c are exact constants. **[6]**

Solution 6

(a)

Mark scheme

- Put $y = e^{-1}$ $(\ln y = -1)$: $-(x^2 - 3) + 6x = 14 \Rightarrow x^2 - 6x + 11 = 0$; discriminant $36 - 44 = -8 < 0$, so there is no real x — no such point (AG).

Solution 6

(b) Mark scheme

- Implicit differentiation: $2x\ln y + \frac{x^2-3}{y}\frac{dy}{dx} + 6 = 0$. At $(2, e^2)$: $8 + \frac{1}{e^2}\frac{dy}{dx} + 6 = 0 \Rightarrow \frac{dy}{dx} = -14e^2$. Tangent: $y = -14e^2x + 29e^2$.