

Learn these by heart before the exam.

## Algebra and functions

$b^2 - 4ac > 0$  two real roots,  $= 0$  one repeated root,  $< 0$  none

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

**Factor and remainder theorem:** if  $f(a) = 0$  then  $(x - a)$  is a factor; dividing  $f(x)$  by  $(x - a)$  leaves remainder  $f(a)$

$$|x| < a \iff -a < x < a$$

$$\log_a x + \log_a y = \log_a(xy)$$

$\log_a x^k = k \log_a x$ ,  $a^x = e^{x \ln a}$  solve  $a^x = b$  by taking logs of both sides

## Sequences and binomial

$$u_n = a + (n - 1)d, \quad S_n = \frac{n}{2}(2a + (n - 1)d)$$

$$u_n = ar^{n-1}, \quad S_n = \frac{a(1 - r^n)}{1 - r}$$

$$S_\infty = \frac{a}{1 - r}, \quad |r| < 1 \quad \text{sum to infinity, converges only when } |r| < 1$$

$$(a + b)^n = \sum_{r=0}^n \binom{n}{r} a^{n-r} b^r$$

$$(1 + x)^n = 1 + nx + \frac{n(n-1)}{2!}x^2 + \dots \quad n \text{ any rational; valid for } |x| < 1$$

## Trigonometry

$$\sin^2 \theta + \cos^2 \theta = 1$$

$$1 + \tan^2 \theta = \sec^2 \theta, \quad 1 + \cot^2 \theta = \csc^2 \theta$$

$$\sin 2\theta = 2 \sin \theta \cos \theta$$

$$\cos 2\theta = 2 \cos^2 \theta - 1 = 1 - 2 \sin^2 \theta$$

$$a \sin \theta + b \cos \theta = R \sin(\theta + \alpha) \quad R = \sqrt{a^2 + b^2}, \quad \tan \alpha = b/a$$

$$\text{arc } s = r\theta, \quad \text{area} = \frac{1}{2}r^2\theta \quad \theta \text{ in radians}$$

## Differentiation and integration

$$\frac{d}{dx}(x^n) = nx^{n-1}$$

$$\frac{d}{dx}(e^{kx}) = ke^{kx}, \quad \frac{d}{dx}(\ln x) = \frac{1}{x}$$

$$\frac{d}{dx}(\sin x) = \cos x, \quad \frac{d}{dx}(\cos x) = -\sin x, \quad \frac{d}{dx}(\tan x) = \sec^2 x$$

$$\text{product: } (uv)' = u'v + uv'$$

$$\text{quotient: } \left(\frac{u}{v}\right)' = \frac{u'v - uv'}{v^2}$$

$$\text{chain: } \frac{dy}{dx} = \frac{dy}{du} \cdot \frac{du}{dx}$$

Stationary point where  $\frac{dy}{dx} = 0$ ;  $\frac{d^2y}{dx^2} > 0$  minimum,  $< 0$

maximum.

$$\int x^n dx = \frac{x^{n+1}}{n+1} + c, \quad n \neq -1$$

$$\int \frac{1}{x} dx = \ln|x| + c, \quad \int e^{kx} dx = \frac{1}{k}e^{kx} + c$$

Area =  $\int_a^b y dx$ ,  $V = \pi \int_a^b y^2 dx$  area under a curve; volume of revolution about the  $x$ -axis

**Trapezium rule:**

$$\int_a^b y dx \approx \frac{h}{2}[y_0 + 2(y_1 + \dots + y_{n-1}) + y_n], \quad h = \frac{b-a}{n}$$

## Vectors and numerical methods

$$|\mathbf{a}| = \sqrt{x^2 + y^2 + z^2}$$

$\mathbf{a} \cdot \mathbf{b} = |\mathbf{a}||\mathbf{b}| \cos \theta = 0$  when the vectors are perpendicular

$\mathbf{r} = \mathbf{a} + t\mathbf{d}$  vector equation of a line through  $\mathbf{a}$ , direction  $\mathbf{d}$

**Newton-Raphson:**  $x_{n+1} = x_n - \frac{f(x_n)}{f'(x_n)}$  - iterate to a root

A sign change of  $f(x)$  over  $[a, b]$  (continuous) locates a root between them.

## Mechanics

$v = u + at$ ,  $s = ut + \frac{1}{2}at^2$ ,  $v^2 = u^2 + 2as$  constant acceleration (suvat) only

$F = ma$  resultant force; resolve forces along and perpendicular to motion

$F \leq \mu R$  friction;  $F = \mu R$  at the point of slipping

$P = Fv$  power = force  $\times$  velocity (driving force)

$$W = Fd \cos \theta, \quad E_k = \frac{1}{2}mv^2, \quad E_p = mgh$$

work-energy: work done = change in kinetic energy

Projectile: horizontal velocity constant; vertical acceleration =  $-g$  (9.8 m/s<sup>2</sup>).

## Statistics

$$\bar{x} = \frac{\sum x}{n}, \quad \sigma^2 = \frac{\sum x^2}{n} - \bar{x}^2$$

$$P(A \cup B) = P(A) + P(B) - P(A \cap B)$$

$P(A \cap B) = P(A)P(B|A)$  independent when

$$P(A \cap B) = P(A)P(B)$$

$X \sim B(n, p)$ :  $P(X = r) = \binom{n}{r}p^r(1-p)^{n-r}$  mean  $np$ , variance  $np(1-p)$

$X \sim N(\mu, \sigma^2)$ :  $Z = \frac{X - \mu}{\sigma}$  standardise, then read  $\Phi(z)$  from tables

Normal approximates  $B(n, p)$  when  $n$  large,  $np > 5$  and  $n(1-p) > 5$  (use a continuity correction).