2. Further Pure Mathematics 2

A-Level Further Mathematics Name: ____________________ Score: ________

1. Using cosh x = (e^x + e^{-x})/2, what is cosh 0?

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2. The matrix [[2,1],[1,2]] has characteristic equation $(2-\lambda)^2 - 1 = 0$, giving $\lambda = 1$ or $\lambda = 3$. What is the larger eigenvalue?

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3. The derivative of sinh x is:

   ☐ cosh x

   ☐ −cosh x

   ☐ sinh x

   ☐ tanh x

4. What is $\int$ 1/(a^2 + x^2) dx?

   ☐ (1/a) $\tan^{-1}$(x/a) + C

   ☐ $\sin^{-1}$(x/a) + C

   ☐ ln(a^2 + x^2) + C

   ☐ a $\tan^{-1}$(ax) + C

5. De Moivre's theorem states (cos θ + i sin θ)^n equals:

   ☐ cos nθ + i sin nθ

   ☐ n(cos θ + i sin θ)

   ☐ cos θ^n + i sin θ^n

   ☐ cos(θ/n) + i sin(θ/n)

6. For dy/dx + 2y = e^x, the integrating factor μ = e^($\int$2 dx) is:

   ☐ e^(2x)

   ☐ e^x

   ☐ 2x

   ☐ e^(x^2)

7. For any x, what does $\cosh^2$x − $\sinh^2$x equal?

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8. The eigenvalues of a matrix A are found by solving:

- ☐ $\det(A - \lambda I) = 0$
- ☐ $A = 0$
- ☐ $\det(A) = 1$
- ☐ $Av = 0$

9. The derivative of $\tan^{-1}x$ is $1/(1+x^2)$. What is its value at $x = 0$?

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10. How many distinct roots of unity does the equation $z^5 = 1$ have?

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A-Level Further Mathematics

1. **1**

   $\cosh 0 = (e^0 + e^0)/2 = (1 + 1)/2 = 1.$

2. **3**

   $(2-\lambda)^2 = 1 \to 2-\lambda = \pm 1 \to \lambda = 1$ or 3; the larger is 3.

3. **cosh x**

   d/dx(sinh x) = cosh x (and d/dx(cosh x) = sinh x).

4. **$(1/a)\tan^{-1}(x/a) + C$**

   This is the standard inverse-tangent integral.

5. **$\cos n\theta + i \sin n\theta$**

   Raising to the power n multiplies the argument by n: $\cos n\theta + i \sin n\theta$.

6. **e^(2x)**

   $\int 2\,dx = 2x$, so $\mu = e\hat{}(2x)$.

7. **1**

   The hyperbolic identity is $\cosh^2 x - \sinh^2 x = 1$.

8. **$\det(A - \lambda I) = 0$**

   The characteristic equation $\det(A - \lambda I) = 0$ gives the eigenvalues.

9. **1**

   $1/(1+0^2) = 1.$

10. **5**

    $z^n = 1$ has exactly n roots, equally spaced on the unit circle; here n = 5.