A-Level Further Mathematics

Formula Sheet · 公式表

MF19 carries a Further Mathematics section: the summation formulae, the Maclaurin series, the hyperbolic identities and the standard statistical tests. It does not carry the matrix methods, the vector geometry results or the differential-equation techniques that most of the paper actually consists of. 公式表含部分进阶内容，但矩阵、向量几何与微分方程方法均需自备。

Given in MF19 – further sections

The summations $\sum r$, $\sum r^2$ and $\sum r^3$ in closed form.

Maclaurin expansions, including those for $\sinh x$, $\cosh x$, $\arctan x$ and $\arcsin x$.

The hyperbolic identities and the logarithmic forms of the inverse hyperbolic functions.

Statistics: the t, χ^2 and F distributions with their tables, and the standard formulae for the correlation coefficient and the regression line.

Mechanics: moments of inertia of standard bodies, and the formulae for elastic strings and simple harmonic motion.

You must know – matrices and linear algebra

Determinant and inverse 行列式与逆矩阵 — For a 2×2 matrix, $\det = ad - bc$ and the inverse is $\frac{1}{ad-bc}\begin{pmatrix} d & -b \\ -c & a \end{pmatrix}$. A zero determinant means singular: no inverse, and the transformation collapses the plane onto a line.

What the determinant means 行列式的几何意义 — Its absolute value is the area (or volume) scale factor of the transformation, and a negative value means orientation is reversed.

Eigenvalues and eigenvectors 特征值与特征向量 — Solve $\det(\mathbf{A} - \lambda\mathbf{I}) = 0$ for λ, then $(\mathbf{A} - \lambda\mathbf{I})\mathbf{x} = \mathbf{0}$ for each eigenvector. Diagonalisation gives $\mathbf{A} = \mathbf{PDP}^{-1}$, and hence $\mathbf{A}^n = \mathbf{PD}^n\mathbf{P}^{-1}$.

Systems of equations 线性方程组 — Row-reduce the augmented matrix. A row of zeros equalling a non-zero constant means inconsistent; a row entirely of zeros means infinitely many solutions, which must then be given in parametric form.

You must know – complex numbers and roots

Modulus-argument form 模辐角形式 — $z = r(\cos\theta + i\sin\theta) = re^{i\theta}$; multiplying multiplies moduli and adds arguments.

De Moivre 棣莫弗定理 — $(re^{i\theta})^n = r^n e^{in\theta}$, which gives both the multiple-angle identities and the nth roots, spaced $2\pi/n$ apart around a circle.

Roots of polynomials 多项式根与系数 — For $ax^3 + bx^2 + cx + d = 0$: $\sum\alpha = -b/a$, $\sum\alpha\beta = c/a$, $\alpha\beta\gamma = -d/a$. New equations with transformed roots come from a substitution, not from solving.

Loci in the complex plane 复平面轨迹 — $|z - a| = r$ is a circle; $|z - a| = |z - b|$ is the perpendicular bisector; $\arg(z - a) = \theta$ is a half-line from a, excluding a itself.

You must know – calculus and differential equations

First-order linear equations 一阶线性微分方程 — Write it as $\frac{dy}{dx} + P(x)y = Q(x)$, multiply by the integrating factor $e^{\int P\,dx}$, then the left side is the derivative of $y\times$ that factor.

Second-order equations 二阶微分方程 — Solve the auxiliary equation; real distinct roots give $Ae^{m_1x} + Be^{m_2x}$, a repeated root gives $(A + Bx)e^{mx}$, complex roots give $e^{px}(A\cos qx + B\sin qx)$. Then add a particular integral matched to the right-hand side.

Reduction formulae 递推公式 — Integrate by parts once to express I_n in terms of I_{n-2}, and state the base case. The exam always asks for the base case separately.

Arc length and surface area 弧长与旋转曲面 — $s = \int \sqrt{1 + (dy/dx)^2}\,dx$, and for a surface of revolution about the x-axis, $S = 2\pi \int y\sqrt{1 + (dy/dx)^2}\,dx$.

Polar coordinates 极坐标 — Area $= \frac{1}{2}\int r^2\,d\theta$. Getting the limits right – where the curve actually starts and finishes – carries more marks than the integration.

You must know – vector geometry

Vector product 向量积 — $|\mathbf{a} \times \mathbf{b}| = |\mathbf{a}||\mathbf{b}|\sin\theta$, giving the area of a parallelogram; the result is perpendicular to both, so it supplies a plane's normal.

Lines and planes 直线与平面 — Line: $\mathbf{r} = \mathbf{a} + t\mathbf{b}$. Plane: $\mathbf{r} \cdot \mathbf{n} = d$. The angle between two planes is the angle between their normals; between a line and a plane it is the complement of that.

Distances 距离 — Point to plane: $|\mathbf{a} \cdot \hat{\mathbf{n}} - d|$. Between skew lines: project the joining vector onto the common perpendicular $\mathbf{b}_1 \times \mathbf{b}_2$.

Where Further Maths marks are won and lost

Method marks dominate. State the technique – "using the integrating factor", "by de Moivre" – before the algebra, because a stated method survives an arithmetic slip.

Proof by induction has a fixed shape and is marked against it: verify the base case, assume for $n = k$, prove for $n = k + 1$, then write the concluding sentence. The conclusion is a mark of its own and is the one most often missing.

Give exact answers throughout unless a decimal is asked for, and check a complex-number argument lies in the required range before writing it down.