4.2 Inference using normal and t-distributio

Name: ____________ Class: ________ Date: __________ **Total: 12 marks**

KEY WORDS confidence interval 置信区间 • degrees of freedom 自由度

Objective

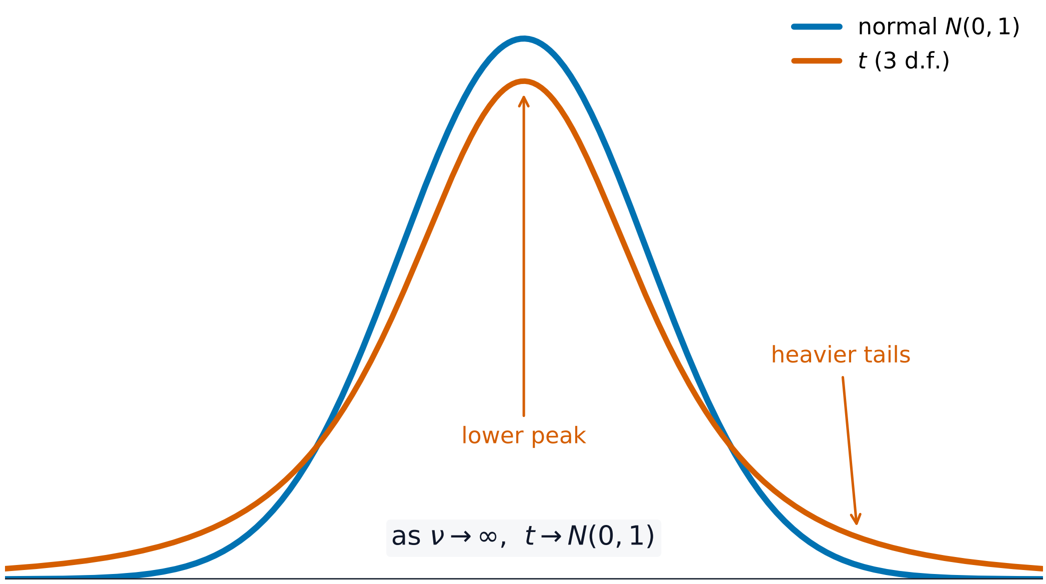
Build the skills to answer exam questions on **inference using the normal and t-distributions** 正态与 t 分布推断 — small-sample **confidence intervals** 置信区间 and **t-tests** t 检验.

You must be able to:

- find a confidence interval $\bar{x} \pm t\dfrac{s}{\sqrt{n}}$ (with $n-1$ d.f.)
- carry out a one-sample t-test
- carry out a two-sample / paired t-test (pooled variance where needed)

1 Worked examples

■ Small-sample interval



For a small sample the t-distribution is flatter with heavier tails, so its critical values are larger

$n = 10,\ \bar{x} = 50,\ s = 4,\ 95\%\ (t = 2.262,\ 9$ **d.f.**$)$: $50 \pm 2.262\dfrac{4}{\sqrt{10}} = 50 \pm 2.86 =$

(47.1, 52.9).

2 Practice

2.1 State the number of degrees of freedom for a one-sample t-test with $n = 12$. [1]

2.2 State when a t-distribution is used instead of the normal. [1]

3 Exam-style questions

3.1 A 95% confidence interval based on a small sample uses a t value that is, compared with 1.96: [1]

- **A** smaller
- **B** larger
- **C** equal
- **D** zero

3.2 A sample of 8 measurements has $\bar{x} = 20.5$ and $s = 1.2$.

(a) Find a 95% confidence interval for the population mean (use $t = 2.365$, 7 d.f.). [3]

(b) State one assumption needed for this interval to be valid. [1]

3.3 A machine should produce rods of mean length 25 cm. A sample of 6 rods has $\bar{x} = 25.4$ cm and $s = 0.5$ cm. Test at the 5% level (two-tailed, $t_{\text{crit}} = 2.571$) whether the mean differs from 25. [5]

Solutions

2.1 $n - 1 = 11$.

2.2 when the sample is small and the population variance is unknown.

3.1 **B**. The small-sample t value is larger than 1.96.

3.2 (a) $20.5 \pm 2.365\dfrac{1.2}{\sqrt{8}} = 20.5 \pm 2.365(0.4243) = 20.5 \pm 1.00$; (19.5, 21.5).
(b) the population is (approximately) normally distributed.

3.3 $H_0 : \mu = 25$, $H_1 : \mu \neq 25$; $t = \dfrac{25.4 - 25}{0.5/\sqrt{6}} = \dfrac{0.4}{0.2041} = 1.96$; compare with 2.571; since $1.96 < 2.571$, do not reject H_0: no evidence the mean differs from 25.