

3.3 Circular motion

Name: _____ Class: _____ Date: _____

Total: 16 marks

KEY WORDS angular speed 角速度 • centripetal acceleration 向心加速度 • vertical circle 竖直圆 • horizontal circle 水平圆
• circular motion 圆周运动

Objective

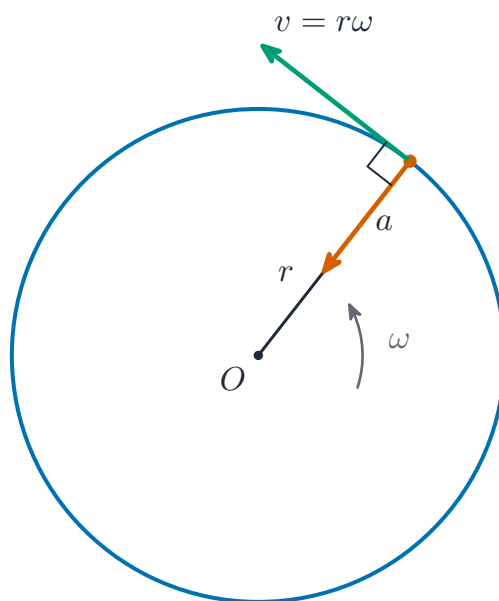
Build the skills to answer exam questions on **circular motion** 圆周运动 — **angular speed** 角速度, **centripetal acceleration** 向心加速度, the **conical pendulum** 圆锥摆, and motion in a **vertical circle** 竖直圆. Take $g = 10 \text{ m s}^{-2}$.

You must be able to:

- use $v = r\omega$ and $a = r\omega^2 = \frac{v^2}{r}$
- analyse a horizontal circle (conical pendulum)
- use energy + the centripetal equation for a vertical circle

1 Worked examples

- Centripetal force



Velocity points along the tangent; acceleration points inward to the centre

$r = 2 \text{ m}$, $\omega = 3 \text{ rad s}^{-1}$: $v = r\omega = 6 \text{ m s}^{-1}$; $a = r\omega^2 = 18 \text{ m s}^{-2}$.

Centripetal force $= ma = \frac{mv^2}{r}$, provided by tension, friction or a component of weight.

2 Practice

2.1 A particle moves in a circle of radius 0.5 m at 4 m s^{-1} . Find its acceleration. [2]

2.2 Convert an angular speed of 120 rev/min to rad s^{-1} . [2]

3 Exam-style questions

3.1 The centripetal acceleration of a particle moving at speed v in a circle of radius r is: [1]

- A $\frac{v}{r}$
- B $\frac{v^2}{r}$

- C vr
- D $\frac{r}{v^2}$

3.2 A particle of mass 0.2 kg is attached to a string of length 0.5 m and whirled in a horizontal circle on a smooth table at 3 rev s⁻¹.

(a) Find the angular speed in rad s⁻¹. [2]

(b) Find the tension in the string. [3]

3.3 A conical pendulum has a bob of mass 0.4 kg on a string of length 1 m, which makes an angle of 30° with the vertical.

(a) Find the tension in the string. [3]

(b) Find the angular speed of the bob. [3]

Solutions

2.1 $a = \frac{v^2}{r} = \frac{16}{0.5} = 32 \text{ m s}^{-2}.$

2.2 $120 \text{ rev/min} = 2 \text{ rev/s} = 2 \times 2\pi = 4\pi = 12.6 \text{ rad s}^{-1}.$

3.1 B. $a = \frac{v^2}{r}$ (also $= r\omega^2$).

3.2 (a) $\omega = 3 \times 2\pi = 6\pi = 18.8 \text{ rad s}^{-1}.$

(b) $T = mr\omega^2 = 0.2 \times 0.5 \times (6\pi)^2 = 0.2 \times 0.5 \times 355.3 = 35.5 \text{ N}.$

3.3 (a) vertical: $T \cos 30^\circ = mg = 4 \Rightarrow T = \frac{4}{0.866} = 4.62 \text{ N}.$

(b) horizontal: $T \sin 30^\circ = mr\omega^2$ with $r = 1 \times \sin 30^\circ = 0.5$; $4.62 \times 0.5 = 0.4 \times 0.5 \times \omega^2 \Rightarrow \omega^2 = 11.55 \Rightarrow \omega = 3.40 \text{ rad s}^{-1}.$