3.1 Motion of a projectile

Name: ____________ Class: ________ Date: __________ **Total: 15 marks**

KEY WORDS motion of a projectile 抛射体运动 • projectile 抛射体 • trajectory 轨迹

Objective

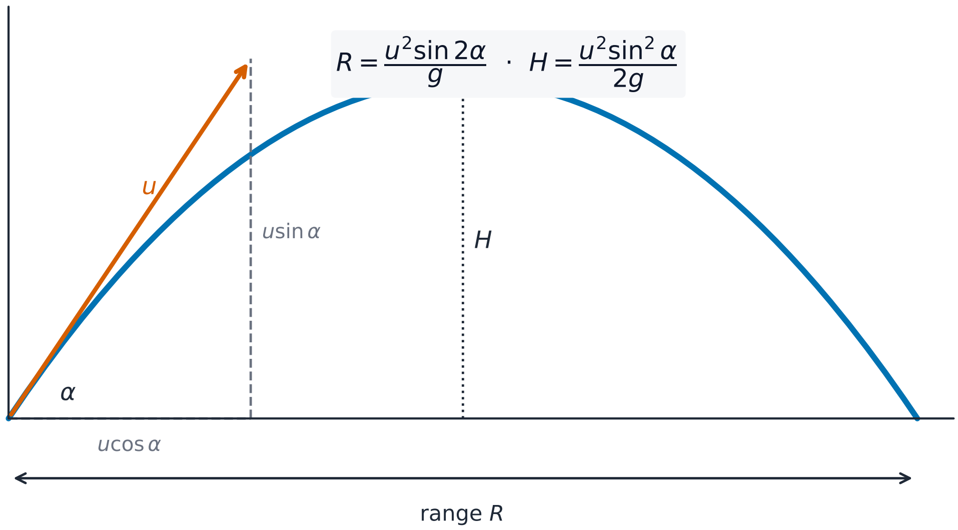
Build the skills to answer exam questions on the **motion of a projectile 抛射体运动** — separating horizontal/vertical motion, the **trajectory 轨迹** equation, range and greatest height. Take $g = 10$ m s^{-2}.

You must be able to:

- use $x = u\cos\alpha\, t$, $y = u\sin\alpha\, t - \frac{1}{2}gt^2$
- eliminate t to get the Cartesian trajectory
- find range $\dfrac{u^2\sin 2\alpha}{g}$ and greatest height $\dfrac{u^2\sin^2\alpha}{2g}$

1 Worked examples

■ Range and height



The path is a parabola; u splits into a steady horizontal part and a vertical part slowed by gravity

$u = 20$, $\alpha = 30°$: range $= \dfrac{400 \sin 60°}{10} = 34.6$ m; greatest height $= \dfrac{400 \sin^2 30°}{20} = 5$ m.

2 Practice

2.1 A projectile is launched at 25 m s^{-1} at 40°. Find its initial vertical velocity component. [2]

2.2 Write down the formula for the range on level ground. [1]

3 Exam-style questions

3.1 A ball is projected horizontally at 15 m s^{-1} from a height. After 2 s its horizontal distance is: [1]

- **A** 15 m
- **B** 30 m
- **C** 20 m
- **D** 7.5 m

3.2 A particle is projected at 30 m s^{-1} at 60° to the horizontal.

(a) Find the time to reach the greatest height. [2]

(b) Find the greatest height reached. [3]

3.3 A stone is thrown at 20 m s^{-1} at 45° from the top of a cliff. It lands in the sea 50 m horizontally away.

(a) Find the time of flight. [3]

(b) Find the height of the cliff above the sea. [3]

Solutions

2.1 $25 \sin 40° = 25 \times 0.643 = 16.1$ m s^{-1}.

2.2 $\dfrac{u^2 \sin 2\alpha}{g}$.

3.1 B. $x = 15 \times 2 = 30$ m.

3.2 (a) at greatest height the vertical velocity is 0: $0 = 30 \sin 60° - 10t \Rightarrow t = \dfrac{25.98}{10} = 2.60$ s.
(b) height $= \dfrac{u^2 \sin^2 \alpha}{2g} = \dfrac{900 \times 0.75}{20} = 33.75$ m.

3.3 (a) horizontal: $50 = 20 \cos 45°\, t = 14.14t \Rightarrow t = 3.54$ s.
(b) take up positive with launch at $y = 0$; at landing $y = -h$: $-h = 20 \sin 45°(3.54) - 5(3.54)^2$; $= 50.0 - 62.5 = -12.5$, so the cliff is $h = 12.5$ m above the sea.