A-LEVEL FURTHER MATHEMATICS • EXERCISE SHEET

2.3 Differentiation (Maclaurin, hyperbolic, parametric)

Name: ____________ Class: ________ Date: __________ **Total: 14 marks**

KEY WORDS maclaurin's series 麦克劳林级数 • hyperbolic functions 双曲函数 • further differentiation 微分

Objective

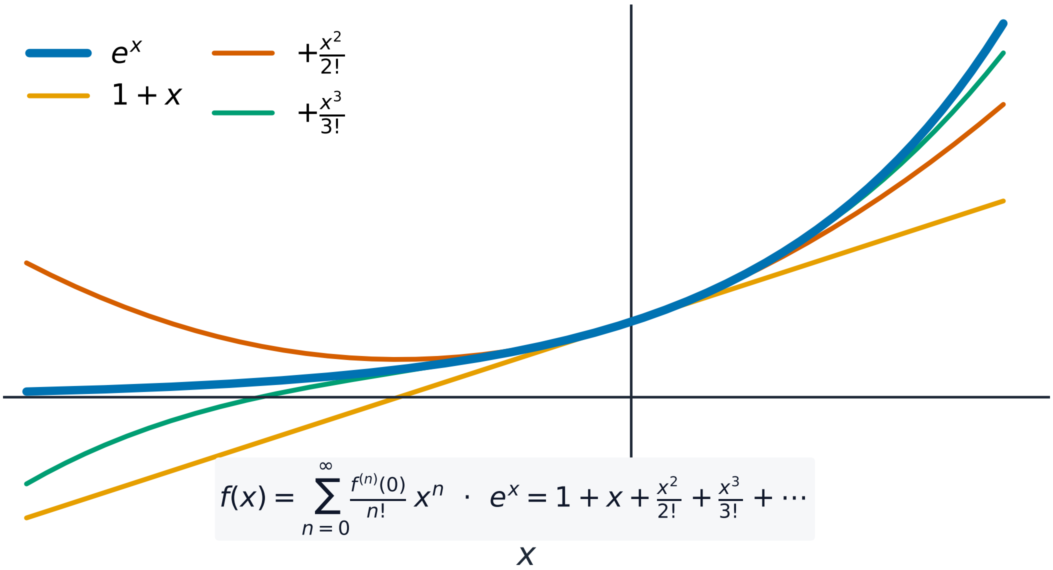
Build the skills to answer exam questions on **further differentiation 微分** — derivatives of inverse and **hyperbolic functions 双曲函数**, second derivatives of implicit/parametric curves, and **Maclaurin's series 麦克劳林级数**.

You must be able to:

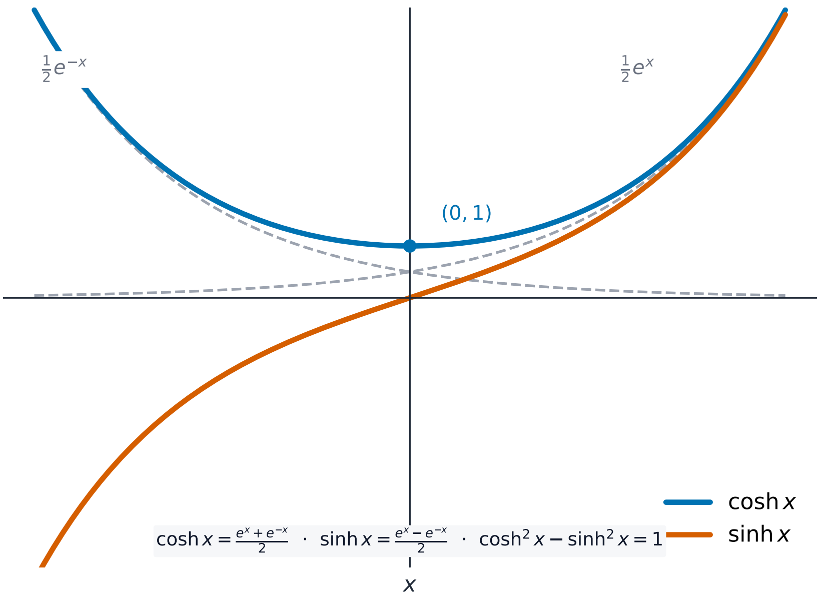
- differentiate $\sin^{-1} x$, $\tan^{-1} x$, $\sinh x$, $\cosh x$, $\tanh x$
- find $\dfrac{d^2y}{dx^2}$ for implicit/parametric curves
- find a Maclaurin series for a function

1 Worked examples

Maclaurin series



Adding more terms makes the polynomial match the function over a wider stretch around $x = 0$



The hyperbolic functions whose derivatives appear here

Maclaurin: $f(x) = f(0) + f'(0)x + \dfrac{f''(0)}{2!}x^2 + \cdots$; e.g. $e^x = 1 + x + \dfrac{x^2}{2!} + \cdots$.

Derivatives: $\dfrac{d}{dx}\sinh x = \cosh x$; $\dfrac{d}{dx}\tan^{-1} x = \dfrac{1}{1+x^2}$.

2 Practice

2.1 Differentiate $y = \cosh(3x)$. [2]

2.2 Differentiate $y = \sin^{-1}(2x)$. [2]

3 Exam-style questions

3.1 $\dfrac{d}{dx}\tanh x$ equals: [1]

- **A** $\operatorname{sech}^2 x$
- **B** $\operatorname{cosech}^2 x$
- **C** $\cosh^2 x$
- **D** $\sinh^2 x$

3.2 Find the Maclaurin series for $\ln(1 + x)$ up to and including the term in x^3. [4]

3.3 A curve is given by $x = t^2$, $y = t^3$. Find $\dfrac{d^2y}{dx^2}$ in terms of t. [5]

Solutions

2.1 $\dfrac{dy}{dx} = 3\sinh(3x)$.

2.2 $\dfrac{dy}{dx} = \dfrac{2}{\sqrt{1-4x^2}}$.

3.1 **A**. $\dfrac{d}{dx}\tanh x = \operatorname{sech}^2 x$.

3.2 $f(0) = 0$; $f'(x) = \dfrac{1}{1+x}$, $f'(0) = 1$; $f''(x) = -\dfrac{1}{(1+x)^2}$, $f''(0) = -1$; $f'''(0) = 2$; $\ln(1+x) = x - \dfrac{x^2}{2} + \dfrac{x^3}{3} - \cdots$.

3.3 $\dfrac{dx}{dt} = 2t$, $\dfrac{dy}{dt} = 3t^2$, so $\dfrac{dy}{dx} = \dfrac{3t^2}{2t} = \dfrac{3t}{2}$; $\dfrac{d^2y}{dx^2} = \dfrac{d}{dt}\left(\dfrac{3t}{2}\right) \div \dfrac{dx}{dt} = \dfrac{3/2}{2t} = \dfrac{3}{4t}$.