

1.6 Vectors

Name: _____ Class: _____ Date: _____

Total: 17 marks

KEY WORDS plane 平面 • vectors 向量 • vector product 向量积

Objective

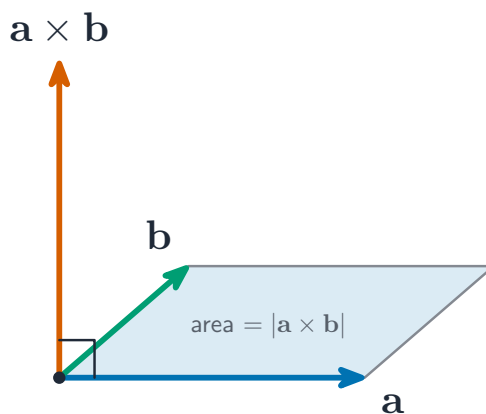
Build the skills to answer exam questions on **vectors** 向量 in 3-D — the **vector product** 向量积, equations of a **plane** 平面, and distances/angles between lines and planes.

You must be able to:

- compute $\mathbf{a} \times \mathbf{b}$ and use it for a normal/area
- write a plane as $\mathbf{r} \cdot \mathbf{n} = p$ or $ax + by + cz = d$
- find the angle between, and distance from, lines and planes

1 Worked examples

■ Vector product & plane



$\mathbf{a} \times \mathbf{b}$ is perpendicular to both; its length is the area of the parallelogram

$\mathbf{a} = \mathbf{i} + \mathbf{j}$, $\mathbf{b} = \mathbf{j} + \mathbf{k}$: $\mathbf{a} \times \mathbf{b} = \mathbf{i} - \mathbf{j} + \mathbf{k}$.

Plane through A with normal $\mathbf{n}$: $\mathbf{r} \cdot \mathbf{n} = \mathbf{a} \cdot \mathbf{n}$.

2 Practice

2.1 Find $\mathbf{a} \times \mathbf{b}$ for $\mathbf{a} = 2\mathbf{i} + \mathbf{j}$, $\mathbf{b} = \mathbf{i} + \mathbf{k}$. [3]

2.2 Write the plane through $(1, 2, 3)$ with normal $\mathbf{n} = (1, 1, 1)$ in the form $ax + by + cz = d$. [2]

3 Exam-style questions

3.1 $\mathbf{a} \times \mathbf{b}$ is a vector that is: [1]

- **A** parallel to $\mathbf{a}$
- **B** perpendicular to both $\mathbf{a}$ and $\mathbf{b}$
- **C** equal to $\mathbf{a} \cdot \mathbf{b}$
- **D** of length $|\mathbf{a}||\mathbf{b}| \cos \theta$

3.2 Points $A(1, 0, 2)$, $B(2, 1, 0)$ and $C(0, 2, 1)$ are given.

(a) Find $\overrightarrow{AB} \times \overrightarrow{AC}$. [4]

(b) Hence find the equation of the plane through A , B , C . [3]

3.3 Find the acute angle between the planes $x + 2y + 2z = 5$ and $2x - y + 2z = 1$. [4]

Solutions

$$\mathbf{2.1} \quad \mathbf{a} \times \mathbf{b} = \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 2 & 1 & 0 \\ 1 & 0 & 1 \end{vmatrix} = \mathbf{i}(1) - \mathbf{j}(2) + \mathbf{k}(-1) = \mathbf{i} - 2\mathbf{j} - \mathbf{k}.$$

$$\mathbf{2.2} \quad x + y + z = 1 + 2 + 3 = 6.$$

3.1 B. The vector product is perpendicular to both vectors.

$$\mathbf{3.2} \quad (\text{a}) \quad \overrightarrow{AB} = (1, 1, -2), \quad \overrightarrow{AC} = (-1, 2, -1); \quad \text{cross} = \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 1 & 1 & -2 \\ -1 & 2 & -1 \end{vmatrix} = \mathbf{i}(-1 + 4) - \mathbf{j}(-1 -$$

$$2) + \mathbf{k}(2 + 1) = (3, 3, 3).$$

(b) normal $(1, 1, 1)$ through $A(1, 0, 2)$: $x + y + z = 3$.

$$\mathbf{3.3} \quad \text{normals } (1, 2, 2) \text{ and } (2, -1, 2); \quad \text{dot} = 2 - 2 + 4 = 4; \quad |\mathbf{n}_1| = 3, \quad |\mathbf{n}_2| = 3; \quad \cos \theta = \frac{4}{9} \Rightarrow \theta = 63.6^\circ.$$