

1.5 Polar coordinates

Name: _____ Class: _____ Date: _____

Total: 15 marks**KEY WORDS** polar coordinates 极坐标 • polar curves 极坐标曲线 • cartesian coordinates 直角坐标

Objective

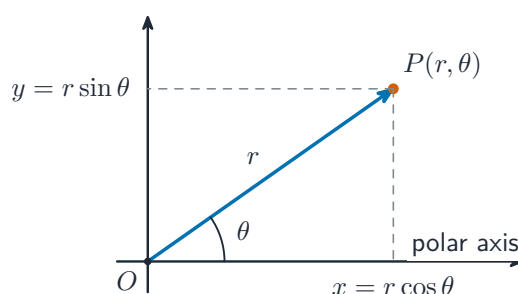
Build the skills to answer exam questions on **polar coordinates** 极坐标 — converting to/from Cartesian, sketching **polar curves** 极坐标曲线, and the **sector area** $\frac{1}{2} \int r^2 d\theta$.

You must be able to:

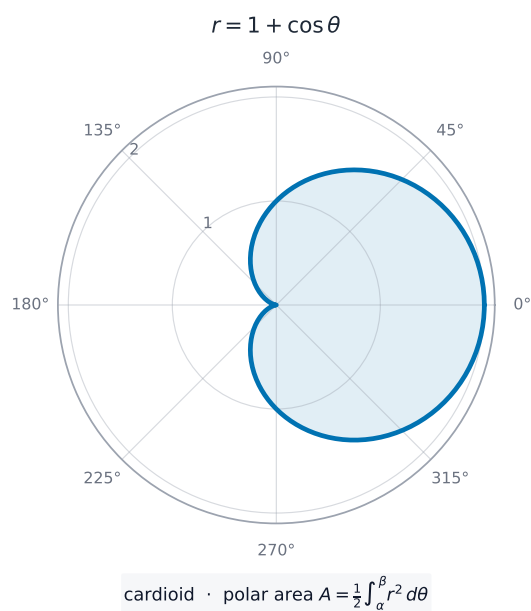
- convert between (r, θ) and (x, y)
- sketch a simple polar curve (e.g. a cardioid)
- find an area with $\frac{1}{2} \int r^2 d\theta$

1 Worked examples

■ Conversion & area



$$x = r \cos \theta, \quad y = r \sin \theta, \quad r^2 = x^2 + y^2$$



Polar curves are traced as the angle varies

Convert $r = 4 \cos \theta$: $\times r \rightarrow r^2 = 4r \cos \theta$, so $x^2 + y^2 = 4x$, i.e. $(x - 2)^2 + y^2 = 4$ (circle).

Area: $A = \frac{1}{2} \int_{\alpha}^{\beta} r^2 d\theta$.

2 Practice

2.1 Convert the point $(r, \theta) = (2, \frac{\pi}{3})$ to Cartesian coordinates. [2]

2.2 Write down the formula for the area of a polar sector. [1]

3 Exam-style questions

3.1 The polar equation $r = 2 \sin \theta$ represents: [1]

- **A** a line
 - **B** a circle
 - **C** a parabola
 - **D** a spiral
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3.2 A curve has polar equation $r = 1 + \cos \theta$ (a cardioid).

(a) Find r when $\theta = 0$ and $\theta = \frac{\pi}{2}$. [2]

(b) Find the area enclosed by the curve, using $A = \frac{1}{2} \int_0^{2\pi} (1 + \cos \theta)^2 d\theta$. [5]

3.3 Convert $r = \frac{2}{1 + \cos \theta}$ to Cartesian form. [4]

Solutions

2.1 $x = 2 \cos \frac{\pi}{3} = 1, y = 2 \sin \frac{\pi}{3} = \sqrt{3}.$

2.2 $A = \frac{1}{2} \int r^2 d\theta.$

3.1 B. $r = 2 \sin \theta \Rightarrow r^2 = 2r \sin \theta \Rightarrow x^2 + y^2 = 2y,$ a circle.

3.2 (a) $\theta = 0: r = 2; \theta = \frac{\pi}{2}: r = 1.$

(b) $(1 + \cos \theta)^2 = 1 + 2 \cos \theta + \cos^2 \theta = \frac{3}{2} + 2 \cos \theta + \frac{1}{2} \cos 2\theta; A = \frac{1}{2} \left[\frac{3}{2} \theta + 2 \sin \theta + \frac{1}{4} \sin 2\theta \right]_0^{2\pi};$
 $= \frac{1}{2} \left(\frac{3}{2} \cdot 2\pi \right) = \frac{3\pi}{2}.$

3.3 $r + r \cos \theta = 2 \Rightarrow r = 2 - x$ (since $r \cos \theta = x$); $r^2 = (2 - x)^2 \Rightarrow x^2 + y^2 = 4 - 4x + x^2;$
 $y^2 = 4 - 4x,$ i.e. $y^2 = -4(x - 1).$