1.4 Matrices

Name: ____________ Class: ________ Date: __________ **Total: 13 marks**

KEY WORDS transformations 变换 • rotation 旋转 • inverse 逆矩阵 • enlargement 放大 • determinant 行列式

Objective

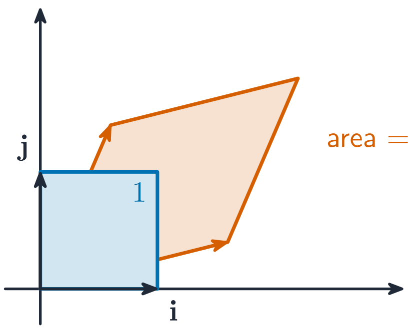
Build the skills to answer exam questions on **matrices** 矩阵 — multiplication, the **determinant** 行列式 and **inverse** 逆矩阵, and 2×2 matrices as **transformations** 变换.

You must be able to:

- multiply matrices and use I
- find $\det A$ and A^{-1} for a 2×2 matrix; use $(AB)^{-1} = B^{-1}A^{-1}$
- identify the transformation a matrix represents (and its area scale factor)

1 Worked examples

■ Inverse & transformation



A matrix maps the unit square to a parallelogram whose area equals $\det A$

$A = \begin{pmatrix} 3 & 1 \\ 2 & 4 \end{pmatrix}$**:** $\det A = 12 - 2 = 10$; $A^{-1} = \frac{1}{10}\begin{pmatrix} 4 & -1 \\ -2 & 3 \end{pmatrix}$.

Transformations: $\begin{pmatrix} 0 & -1 \\ 1 & 0 \end{pmatrix}$ is a $90°$ rotation; $\det = 1$ (area preserved).

2 Practice

2.1 Find $\det \begin{pmatrix} 5 & 2 \\ 3 & 4 \end{pmatrix}$. [1]

2.2 Find the inverse of $\begin{pmatrix} 2 & 1 \\ 1 & 1 \end{pmatrix}$. [2]

3 Exam-style questions

3.1 The matrix $\begin{pmatrix} 2 & 0 \\ 0 & 2 \end{pmatrix}$ represents: [1]

- **A** a rotation
- **B** an enlargement, scale factor 2
- **C** a reflection
- **D** a shear

3.2 $A = \begin{pmatrix} 1 & 2 \\ 3 & 4 \end{pmatrix}$ and $B = \begin{pmatrix} 2 & 0 \\ 1 & 3 \end{pmatrix}$.

(a) Find AB. [2]

(b) Find $\det(AB)$ and verify it equals $\det A \times \det B$. [3]

3.3 The transformation T has matrix $M = \begin{pmatrix} 3 & 1 \\ 0 & 2 \end{pmatrix}$.

(a) Find the area scale factor of T. [1]

(b) Find M^{-1} and hence the image-to-object matrix that undoes T. [3]

Solutions

2.1 det $= 5(4) - 2(3) = 14$.

2.2 det $= 2 - 1 = 1$; inverse $= \begin{pmatrix} 1 & -1 \\ -1 & 2 \end{pmatrix}$.

3.1 B. A scalar multiple of I is an enlargement; here scale factor 2.

3.2 (a) $AB = \begin{pmatrix} 1\cdot 2 + 2\cdot 1 & 1\cdot 0 + 2\cdot 3 \\ 3\cdot 2 + 4\cdot 1 & 3\cdot 0 + 4\cdot 3 \end{pmatrix} = \begin{pmatrix} 4 & 6 \\ 10 & 12 \end{pmatrix}$.
(b) $\det(AB) = 48 - 60 = -12$; $\det A = -2$, $\det B = 6$, product $= -12$ ✓.

3.3 (a) area scale factor $= \det M = 6$.
(b) $M^{-1} = \dfrac{1}{6}\begin{pmatrix} 2 & -1 \\ 0 & 3 \end{pmatrix}$; this matrix undoes T.