A-LEVEL FURTHER MATHEMATICS • EXERCISE SHEET

1.1 Roots of polynomial equations

Name: ______________ Class: __________ Date: ____________ **Total: 15 marks**

KEY WORDS roots 根 • coefficients 系数 • symmetric functions 对称函数 • roots of polynomial equations 多项式方 • substitution 代换

Objective

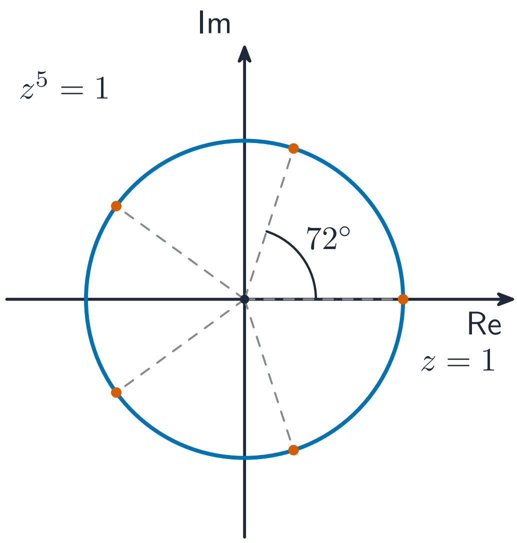
Build the skills to answer exam questions on **roots of polynomial equations** 多项式方程的根 — the relations between roots and **coefficients** 系数, **symmetric functions** 对称函数, and forming a new equation by **substitution** 代换.

You must be able to:

- use $\sum \alpha$, $\sum \alpha\beta$, $\alpha\beta\gamma$ for quadratics and cubics
- evaluate symmetric functions such as $\sum \alpha^2$
- form an equation whose roots are a transformation of the originals

1 Worked examples

■ Roots and coefficients



The roots of a polynomial are tied to its coefficients by $\sum \alpha$, $\sum \alpha\beta$, …

$x^2 - 5x + 6 = 0$ has roots α, β. Find the equation with roots $\alpha + 1, \beta + 1$.
$\alpha + \beta = 5$, $\alpha\beta = 6$; new sum $= 7$, new product $= \alpha\beta + \alpha + \beta + 1 = 12$. So

> $x^2 - 7x + 12 = 0$.
> $\sum \alpha^2 = (\sum \alpha)^2 - 2\sum \alpha\beta$.

2 Practice

2.1 For $x^2 - 7x + 10 = 0$ with roots α, β, write down $\alpha + \beta$ and $\alpha\beta$. [2]

2.2 Hence find $\alpha^2 + \beta^2$. [2]

3 Exam-style questions

3.1 For $2x^3 - 3x^2 + 4x - 5 = 0$ with roots α, β, γ, the value of $\alpha + \beta + \gamma$ is: [1]

- **A** $\frac{3}{2}$
- **B** $-\frac{3}{2}$
- **C** 2
- **D** $\frac{5}{2}$

3.2 The cubic $x^3 - 4x^2 + 5x - 2 = 0$ has roots α, β, γ.

(a) Write down $\sum \alpha$, $\sum \alpha\beta$ and $\alpha\beta\gamma$. [3]

(b) Find $\alpha^2 + \beta^2 + \gamma^2$. [3]

3.3 The equation $x^2 + 3x - 1 = 0$ has roots α, β. Find a quadratic equation with integer coefficients whose roots are $\frac{1}{\alpha}$ and $\frac{1}{\beta}$. [4]

Solutions

2.1 $\alpha + \beta = 7$, $\alpha\beta = 10$.

2.2 $\alpha^2 + \beta^2 = (\alpha + \beta)^2 - 2\alpha\beta = 49 - 20 = 29$.

3.1 **A**. $\alpha + \beta + \gamma = -\dfrac{-3}{2} = \dfrac{3}{2}$.

3.2 (a) $\sum \alpha = 4$, $\sum \alpha\beta = 5$, $\alpha\beta\gamma = 2$.
(b) $\sum \alpha^2 = (\sum \alpha)^2 - 2\sum \alpha\beta = 16 - 10 = 6$.

3.3 $\dfrac{1}{\alpha} + \dfrac{1}{\beta} = \dfrac{\alpha + \beta}{\alpha\beta} = \dfrac{-3}{-1} = 3$; $\dfrac{1}{\alpha\beta} = \dfrac{1}{-1} = -1$; new equation $x^2 - 3x - 1 = 0$.