Exercise walk-through

Non-parametric tests ▪ 4.4

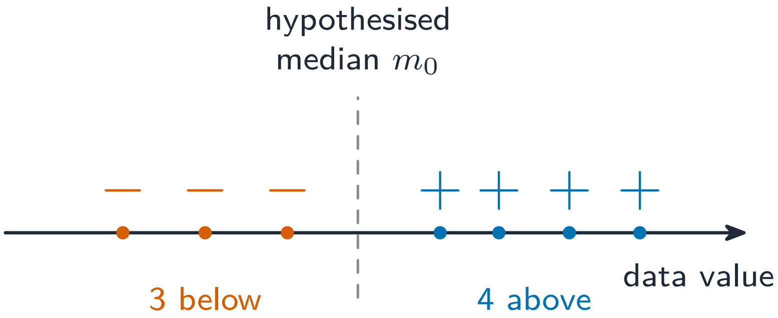
eXercise

You must be able to

- carry out a sign test using a binomial model
- carry out a Wilcoxon signed-rank test on paired data
- choose the right non-parametric test and state when to use one

Method — The sign test

Sign test for median $= m_0$: count $+$ and $-$ signs; under H_0 the number of $+$ is $\sim B(n, 0.5)$; a very lopsided count is significant.



A number line with a dashed median: 3 points below (minus), 4 above (plus)

Practice 2.1 ▪ 1 mark

State when a non-parametric test is preferred over a t-test.

Solution 2.1

Worked steps

when the data cannot be assumed to be normally distributed **B1**.

Practice 2.2 ▪ 1 mark

In a sign test, state the distribution of the number of positive signs under H_0.

Solution 2.2

Method: The sign test

Worked steps

$\sim B(n, 0.5)$ **B1**.

Exam-style 3.1 ▪ 1 mark

The Wilcoxon signed-rank test uses, in addition to the signs of the differences:

A nothing else

B the sizes (ranks) of the differences

C the sample mean

D the population variance

Solution 3.1

Method: The sign test

A nothing else

B the sizes (ranks) of the differences ✓

C the sample mean

D the population variance

Worked steps

B. The signed-rank test also uses the ranks (sizes) of the differences.

Exam-style 3.2 ▪ 2 marks

Ten people rate a product before and after a change. Eight rate it higher $(+)$ and two lower $(-)$.
(a) State H_0 and H_1 for a sign test of “no change” vs “an improvement”.
(b) Using $X \sim B(10, 0.5)$, find $P(X \geqslant 8)$ and state the conclusion at the 5% level.

Solution 3.2

Method: The sign test

Worked steps

(a) H_0: median difference $= 0$ (no change); H_1: median difference > 0 (improvement) **B1** **B1**.

(b) $P(X \geqslant 8) = P(8)+P(9)+P(10) = \dfrac{45+10+1}{1024} = \dfrac{56}{1024} = 0.0547$ **M1** **M1** **M1**; $0.0547 > 0.05$, so do not reject H_0 — insufficient evidence of improvement **A1**.

Exam-style 3.3 ▪ 2 marks

Explain why a non-parametric test might be chosen instead of a t-test for a small sample.

Solution 3.3

Worked steps

a t-test assumes the population is normal; for a small sample that assumption cannot be checked reliably **M1**; a non-parametric test makes no such assumption, so it is safer when normality is doubtful **A1**.