Exercise walk-through

Chi-squared tests ▪ 4.3

eXercise

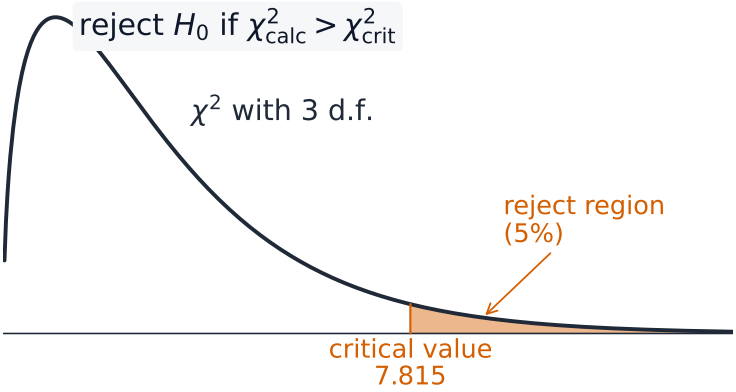
You must be able to

- compute expected counts and $\chi^2 = \sum \frac{(O-E)^2}{E}$
- find the degrees of freedom and compare with the critical value
- carry out a goodness-of-fit and an independence test

Method — Goodness of fit

Four equal categories, observed $20, 30, 25, 25$ (each $E = 25$):
$\chi^2 = \dfrac{25 + 25 + 0 + 0}{25} = 2$. With 3 d.f. the 5% value is 7.815; $2 < 7.815 \rightarrow$ do not reject.

Method — Goodness of fit



A right-skewed chi-squared curve with the upper 5% tail shaded

Practice 2.1 ▪ 1 mark

Write down the χ^2 test statistic formula.

Solution 2.1

Worked steps

$$\chi^2 = \sum \frac{(O-E)^2}{E}$$ B1.

Practice 2.2 ▪ 1 mark

A goodness-of-fit test has 5 categories and no estimated parameters. State the degrees of freedom.

Solution 2.2

Method: Goodness of fit

Worked steps

$5 - 1 = 4$ **B1**.

Exam-style 3.1 ▪ 1 mark

In a χ^2 test, a **large** value of χ^2 suggests:

A observed and expected agree well

B observed and expected differ a lot

C the sample is small

D the test is invalid

Solution 3.1

A observed and expected agree well

B observed and expected differ a lot ✓

C the sample is small

D the test is invalid

Worked steps

B. A large χ^2 means observed counts differ greatly from expected.

Exam-style 3.2 ▪ 6 marks

A die is rolled 120 times, giving the counts:

Face	**1**	**2**	**3**	**4**	**5**	**6**
Observed	15	22	23	19	18	23

Test at the 5% level whether the die is fair ($\chi^2_{\text{crit}} = 11.07$, 5 d.f.).

Solution 3.2

Method: Goodness of fit

Worked steps

each $E = \frac{120}{6} = 20$ M1; $\chi^2 = \frac{(15-20)^2 + (22-20)^2 + (23-20)^2 + (19-20)^2 + (18-20)^2 +$

M1 M1; $= \frac{25+4+9+1+4+9}{20} = \frac{52}{20} = 2.6$ M1; $2.6 < 11.07$ M1; do not reject — no evidence the die is unfair A1.

Exam-style 3.3 ▪ 2 marks

For a 3×4 contingency table, state the number of degrees of freedom for a test of independence.

Solution 3.3

Worked steps

degrees of freedom $= (3-1)(4-1) = 6$ M1 A1.