Exercise walk-through

Motion of a projectile ▪ 3.1

eXercise

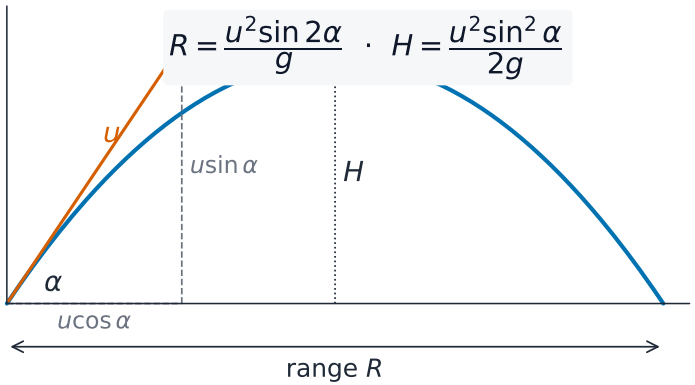
You must be able to

- use $x = u\cos\alpha\, t$, $y = u\sin\alpha\, t - \frac{1}{2}gt^2$
- eliminate t to get the Cartesian trajectory
- find range $\dfrac{u^2\sin 2\alpha}{g}$ and greatest height $\dfrac{u^2\sin^2\alpha}{2g}$

Method — Range and height

$u = 20$, $\alpha = 30^\circ$: range $= \dfrac{400 \sin 60^\circ}{10} = 34.6$ m; greatest height $= \dfrac{400 \sin^2 30^\circ}{20} = 5$ m.

Method — Range and height



A parabolic path with the launch velocity split into horizontal and vertical parts

Practice 2.1 ▪ 2 marks

A projectile is launched at 25 m s^{-1} at $40°$. Find its initial vertical velocity component.

Solution 2.1

Method: Range and height

Worked steps

$25 \sin 40^\circ = 25 \times 0.643 = 16.1$ m s^{-1} M1 A1.

Practice 2.2 ▪ 1 mark

Write down the formula for the range on level ground.

Solution 2.2

Method: Range and height

Worked steps

$$\frac{u^2 \sin 2\alpha}{g}$$ B1.

Exam-style 3.1 ▪ 1 mark

A ball is projected horizontally at 15 m s^{-1} from a height. After 2 s its horizontal distance is:

A 15 m

B 30 m

C 20 m

D 7.5 m

Solution 3.1

Method: Range and height

A 15 m

B 30 m ✓

C 20 m

D 7.5 m

Worked steps

B. $x = 15 \times 2 = 30$ m.

Exam-style 3.2 ▪ 2 marks

A particle is projected at 30 m s^{-1} at 60° to the horizontal.
(a) Find the time to reach the greatest height.
(b) Find the greatest height reached.

Solution 3.2

Method: Range and height

Worked steps

(a) at greatest height the vertical velocity is 0: $0 = 30\sin 60° - 10t \Rightarrow t = \dfrac{25.98}{10} = 2.60$ s M1 A1.

(b) height $= \dfrac{u^2 \sin^2 \alpha}{2g} = \dfrac{900 \times 0.75}{20} = 33.75$ m M1 M1 A1.

Exam-style 3.3 ▪ 3 marks

A stone is thrown at 20 m s^{-1} at 45° from the top of a cliff. It lands in the sea 50 m horizontally away.

(a) Find the time of flight.

(b) Find the height of the cliff above the sea.

Solution 3.3

Method: Range and height

Worked steps

(a) horizontal: $50 = 20\cos 45^\circ\, t = 14.14t \Rightarrow t = 3.54$ s [M1] [M1] [A1].
(b) take up positive with launch at $y = 0$; at landing $y = -h$: $-h = 20\sin 45^\circ(3.54) - 5(3.54)^2$ [M1]; $= 50.0 - 62.5 = -12.5$, so the cliff is $h = 12.5$ m above the sea [M1] [A1].