Exercise walk-through

Differential equations ▪ 2.6

eXercise

You must be able to

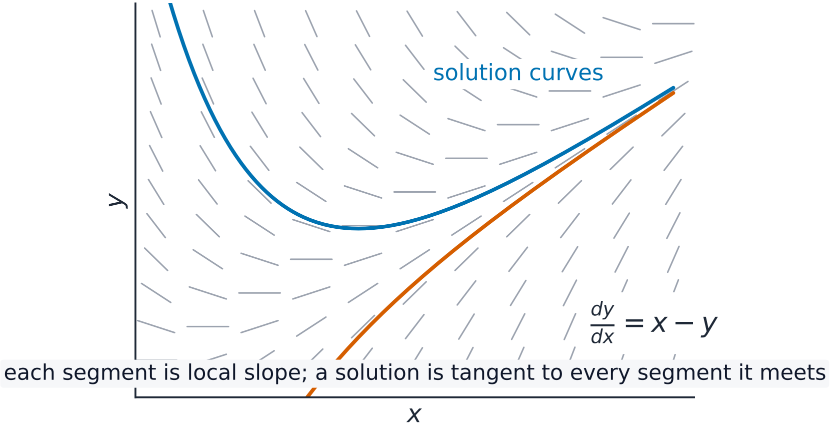
- solve $\dfrac{dy}{dx} + P(x)y = Q(x)$ with $\mu = e^{\int P\,dx}$
- solve $ay'' + by' + cy = 0$ via the auxiliary equation (the CF)
- find a particular integral and the general solution

Method — Integrating factor & CF+PI

$\dfrac{dy}{dx} + 2y = e^x$**:** $\mu = e^{2x}$; $\dfrac{d}{dx}(ye^{2x}) = e^{3x}$, so $y = \frac{1}{3}e^x + Ce^{-2x}$.

Second order: $y'' - 5y' + 6y = 0 \to$ auxiliary $m^2 - 5m + 6 = 0 \to y = Ae^{2x} + Be^{3x}$.

Method — Integrating factor & CF+PI



A slope field with solution curves following the little segments

Practice 2.1 ▪ 1 mark

Write down the integrating factor for $\dfrac{dy}{dx} + 3y = x$.

Solution 2.1

Method: Integrating factor & CF+PI

Worked steps

$\mu = e^{\int 3\,dx} = e^{3x}$ **B1**.

Practice 2.2 ▪ 1 mark

Write the auxiliary equation of $y'' - y' - 6y = 0$.

Solution 2.2

Worked steps

$m^2 - m - 6 = 0$ **B1**.

Exam-style 3.1 ▪ 1 mark

The complementary function of $y'' + 4y = 0$ is:

A $Ae^{2x} + Be^{-2x}$

B $A\cos 2x + B\sin 2x$

C Ae^{2x}

D $A + Bx$

Solution 3.1

- **A** $Ae^{2x} + Be^{-2x}$
- **B** $A\cos 2x + B\sin 2x$ ✓
- **C** Ae^{2x}
- **D** $A + Bx$

Worked steps

B. Auxiliary $m^2 + 4 = 0 \Rightarrow m = \pm 2i$, giving $A\cos 2x + B\sin 2x$.

Exam-style 3.2 ▪ 5 marks

Solve $\dfrac{dy}{dx} + \dfrac{1}{x}y = x^2$ (for $x > 0$), given $y = 1$ when $x = 1$.

Solution 3.2

Worked steps

$\mu = e^{\int (1/x)\,dx} = e^{\ln x} = x$ **M1**; $\dfrac{d}{dx}(xy) = x^3$ **M1**; $xy = \frac{1}{4}x^4 + C$ **M1**; $y = 1$ at $x = 1$: $1 = \frac{1}{4} + C \Rightarrow C = \frac{3}{4}$ **M1**; $y = \frac{1}{4}x^3 + \dfrac{3}{4x}$ **A1**.

Exam-style 3.3 ▪ 6 marks

Solve $\dfrac{d^2y}{dx^2} - 3\dfrac{dy}{dx} + 2y = 4x$.

Solution 3.3

Worked steps

auxiliary $m^2 - 3m + 2 = 0 \Rightarrow m = 1, 2$, so CF $= Ae^x + Be^{2x}$ M1 M1; try PI $y = ax + b$: $-3a + 2(ax + b) = 4x \Rightarrow 2a = 4,\ -3a + 2b = 0$ M1; $a = 2,\ b = 3$ M1; general solution $y = Ae^x + Be^{2x} + 2x + 3$ M1 A1.