Exercise walk-through

Differentiation ▪ 2.3

eXercise

You must be able to

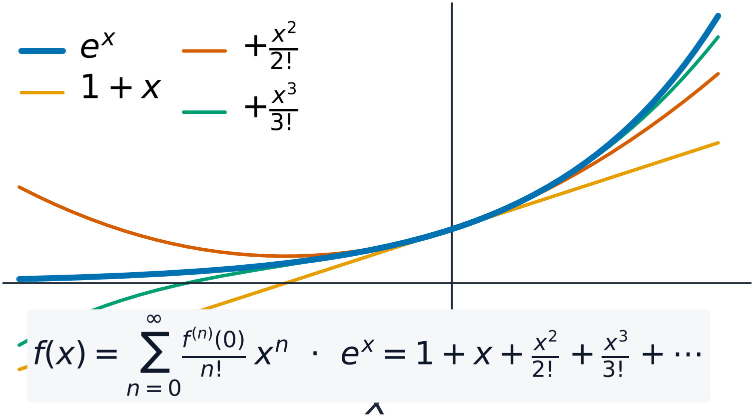
- differentiate $\sin^{-1} x$, $\tan^{-1} x$, $\sinh x$, $\cosh x$, $\tanh x$
- find $\dfrac{d^2y}{dx^2}$ for implicit/parametric curves
- find a Maclaurin series for a function

Method — Maclaurin series

Maclaurin: $f(x) = f(0) + f'(0)x + \dfrac{f''(0)}{2!}x^2 + \cdots$; e.g. $e^x = 1 + x + \dfrac{x^2}{2!} + \cdots$.

Derivatives: $\dfrac{d}{dx}\sinh x = \cosh x$; $\dfrac{d}{dx}\tan^{-1} x = \dfrac{1}{1+x^2}$.

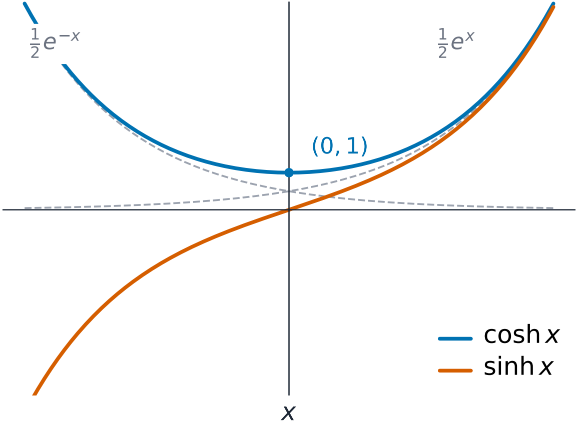
Method — Maclaurin series



$$f(x) = \sum_{n=0}^{\infty} \frac{f^{(n)}(0)}{n!} x^n \quad \cdot \quad e^x = 1 + x + \frac{x^2}{2!} + \frac{x^3}{3!} + \cdots$$

The curve of e^x with polynomial approximations of rising degree

Method — Maclaurin series



The hyperbolic functions whose derivatives appear here

Practice 2.1 ▪ 2 marks

Differentiate $y = \cosh(3x)$.

Solution 2.1

Worked steps

$$\frac{dy}{dx} = 3\sinh(3x) \quad \text{M1}\,\text{A1}.$$

Practice 2.2 ▪ 2 marks

Differentiate $y = \sin^{-1}(2x)$.

Solution 2.2

Worked steps

$$\frac{dy}{dx} = \frac{2}{\sqrt{1-4x^2}}$$ **M1** **A1**.

Exam-style 3.1 ▪ 1 mark

$\dfrac{d}{dx}\tanh x$ equals:

A $\operatorname{sech}^2 x$

B $\operatorname{cosech}^2 x$

C $\cosh^2 x$

D $\sinh^2 x$

Solution 3.1

A $\operatorname{sech}^2 x$ ✓

B $\operatorname{cosech}^2 x$

C $\cosh^2 x$

D $\sinh^2 x$

Worked steps

A. $\dfrac{d}{dx}\tanh x = \operatorname{sech}^2 x$.

Exam-style 3.2 ▪ 4 marks

Find the Maclaurin series for $\ln(1 + x)$ up to and including the term in x^3.

Solution 3.2

Method: Maclaurin series

Worked steps

$f(0) = 0;\ f'(x) = \dfrac{1}{1+x},\ f'(0) = 1;\ f''(x) = -\dfrac{1}{(1+x)^2},\ f''(0) = -1;\ f'''(0) = 2$

M1 **M1**; $\ln(1+x) = x - \dfrac{x^2}{2} + \dfrac{x^3}{3} - \cdots$ **M1** **A1**.

Exam-style 3.3 ▪ 5 marks

A curve is given by $x = t^2$, $y = t^3$. Find $\dfrac{d^2y}{dx^2}$ in terms of t.

Solution 3.3

Method: Maclaurin series

Worked steps

$\dfrac{dx}{dt} = 2t$, $\dfrac{dy}{dt} = 3t^2$, so $\dfrac{dy}{dx} = \dfrac{3t^2}{2t} = \dfrac{3t}{2}$ **M1** **A1**; $\dfrac{d^2y}{dx^2} = \dfrac{d}{dt}\left(\dfrac{3t}{2}\right) \div \dfrac{dx}{dt} = \dfrac{3/2}{2t} = \dfrac{3}{4t}$ **M1** **M1** **A1**.