Exercise walk-through

Hyperbolic functions ▪ 2.1

eXercise

You must be able to

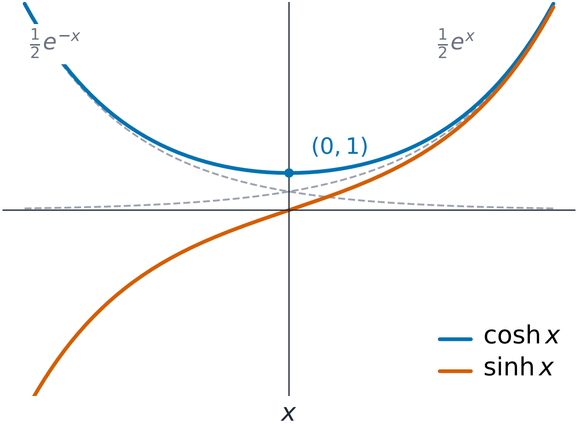
- use $\cosh x = \frac{1}{2}(e^x + e^{-x})$, $\sinh x = \frac{1}{2}(e^x - e^{-x})$
- use $\cosh^2 x - \sinh^2 x = 1$ and related identities
- use $\sinh^{-1} x = \ln(x + \sqrt{x^2 + 1})$ and solve equations

Method — Identity & solving

$$\cosh^2 x - \sinh^2 x = \frac{(e^x + e^{-x})^2 - (e^x - e^{-x})^2}{4} = \frac{4}{4} = 1.$$

Solve $\cosh x = 2$**:** $\dfrac{e^x + e^{-x}}{2} = 2 \Rightarrow e^{2x} - 4e^x + 1 = 0 \Rightarrow e^x = 2 \pm \sqrt{3},\ x = \ln(2 \pm \sqrt{3}).$

Method — Identity & solving



The cosh and sinh curves built from the two exponentials

Practice 2.1 ▪ 1 mark

Write down the value of $\cosh 0$.

Solution 2.1

Worked steps

$\cosh 0 = \frac{1}{2}(1+1) = 1$ **B1**.

Practice 2.2 ▪ 1 mark

Evaluate $\sinh^{-1} 0$ using the logarithmic form.

Solution 2.2

Worked steps

$\sinh^{-1} 0 = \ln(0 + \sqrt{0 + 1}) = \ln 1 = 0$ **B1**.

Exam-style 3.1 ▪ 1 mark

The identity satisfied by hyperbolic functions is:

A $\cosh^2 x + \sinh^2 x = 1$

B $\cosh^2 x - \sinh^2 x = 1$

C $\sinh^2 x - \cosh^2 x = 1$

D $\cosh^2 x - \sinh^2 x = -1$

Solution 3.1

Method: Identity & solving

A $\cosh^2 x + \sinh^2 x = 1$

B $\cosh^2 x - \sinh^2 x = 1$ ✓

C $\sinh^2 x - \cosh^2 x = 1$

D $\cosh^2 x - \sinh^2 x = -1$

Worked steps

B. $\cosh^2 x - \sinh^2 x = 1$.

Exam-style 3.2 ▪ 4 marks

Solve the equation $\cosh x = 3$, giving exact answers.

Solution 3.2

Method: Identity & solving

Worked steps

$\dfrac{e^x + e^{-x}}{2} = 3 \Rightarrow e^{2x} - 6e^x + 1 = 0$ **M1**; $e^x = \dfrac{6 \pm \sqrt{32}}{2} = 3 \pm 2\sqrt{2}$ **M1** **M1**; $x = \ln(3 \pm 2\sqrt{2})$ **A1**.

Exam-style 3.3 ▪ 4 marks

(a) Show that $\sinh^{-1} x = \ln(x + \sqrt{x^2 + 1})$.
(b) Hence evaluate $\sinh^{-1}(\frac{3}{4})$ as a single logarithm.

Solution 3.3

Method: Identity & solving

Worked steps

(a) let $y = \sinh^{-1} x$, so $x = \sinh y = \frac{1}{2}(e^y - e^{-y})$ **M1**; $e^{2y} - 2xe^y - 1 = 0 \Rightarrow e^y = x + \sqrt{x^2+1}$ (positive root) **M1** **M1**; $y = \ln(x + \sqrt{x^2+1})$ **A1**.

(b) $\sinh^{-1} \frac{3}{4} = \ln\left(\frac{3}{4} + \sqrt{\frac{9}{16}+1}\right) = \ln\left(\frac{3}{4} + \frac{5}{4}\right) = \ln 2$ **M1** **A1**.