

Exercise walk-through

Proof by induction ■ 1.7

eXercise

You must be able to

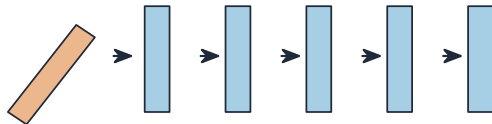
- prove a summation formula by induction
- prove a divisibility result by induction
- write the base case, inductive step and conclusion clearly

Method — The two steps

Prove $\sum_{r=1}^n r = \frac{1}{2}n(n+1)$. Base $n = 1$: $1 = \frac{1}{2}(1)(2)$. Assume true for k ; then $\sum_1^{k+1} = \frac{1}{2}k(k+1) + (k+1) = \frac{1}{2}(k+1)(k+2)$ — the formula at $n = k+1$. So true for all n .

Method — The two steps

so every domino falls — true for all n



base case
inductive step: each knocks the next
first falls

A row of dominoes: the first falls, each knocks the next

Practice 2.1 ■ 2 marks

State the two steps of a proof by induction.

Solution 2.1

Method: The two steps

Worked steps

base case (true for $n = 1$) **B1**; inductive step (assume $n = k$, prove $n = k + 1$)
B1.

Practice 2.2 ■ 1 mark

Verify the base case $n = 1$ for $\sum_{r=1}^n r^2 = \frac{1}{6}n(n+1)(2n+1)$.

Solution 2.2

Method: The two steps

Worked steps

$$\text{LHS} = 1, \text{ RHS} = \frac{1}{6}(1)(2)(3) = 1 \quad \text{B1}.$$

Exam-style 3.1 ■ 1 mark

In an inductive proof, the inductive step assumes the result for $n = k$ and proves it for:

A $n = 1$

B $n = k - 1$

C $n = k + 1$

D all n at once

Solution 3.1

Method: The two steps

A $n = 1$

B $n = k - 1$

C $n = k + 1$



D all n at once

Worked steps

C. The inductive step proves the result for $n = k + 1$.

Exam-style 3.2 ■ 5 marks

Prove by induction that $\sum_{r=1}^n r(r+1) = \frac{1}{3}n(n+1)(n+2)$.

Solution 3.2

Method: The two steps

Worked steps

base $n = 1$: LHS = 2, RHS = $\frac{1}{3}(1)(2)(3) = 2$ **B1**; assume $\sum_1^k r(r+1) = \frac{1}{3}k(k+1)(k+2)$ **M1**; $\sum_1^{k+1} = \frac{1}{3}k(k+1)(k+2) + (k+1)(k+2)$ **M1**; $= (k+1)(k+2)(\frac{k}{3} + 1) = \frac{1}{3}(k+1)(k+2)(k+3)$ **M1**; this is the formula at $n = k+1$, so true for all n **A1**.

Exam-style 3.3 ■ 5 marks

Prove by induction that $7^n - 1$ is divisible by 6 for all positive integers n .

Solution 3.3

Method: The two steps

Worked steps

base $n = 1$: $7^1 - 1 = 6$, divisible by 6 (B1); assume $7^k - 1 = 6m$ for integer m (M1); $7^{k+1} - 1 = 7 \cdot 7^k - 1 = 7(6m + 1) - 1 = 42m + 6 = 6(7m + 1)$ (M1) (M1); divisible by 6, so by induction true for all n (A1).