Past-paper walk-through

Vectors ▪ 1.6

eXercise

Questions in this set

- 9231/11 N25 Q5 ▪ 11 marks
- 9231/12 N25 Q6 ▪ 14 marks
- 9231/14 N25 Q4 ▪ 10 marks
- 9231/11 J25 Q6 ▪ 16 marks
- 9231/13 J25 Q5 ▪ 11 marks

Reading the mark scheme

- **B1** a mark that stands on its own – the point is either made or not.
- **M1** a *method* mark: the right approach, even if the number is wrong.
- **A1** an *answer* mark, and it usually needs its M mark first.
- **C1** a working mark on the way to the answer.
- // separates answers the examiner accepts equally.
- **ecf** = error carried forward: a later part is marked on your own earlier answer.
- **owtte** = "or words to that effect" – the wording need not match.

Question 5

9231/11 N25 ▪ Q5 ▪ 11 marks

The plane Π_1 has equation $\mathbf{r} = -3\mathbf{i} - \mathbf{j} - \mathbf{k} + \lambda(\mathbf{j} + 2\mathbf{k}) + \mu(\mathbf{i} + 3\mathbf{j} + \mathbf{k})$.

(a) Find an equation for Π_1 in the form $ax + by + cz = d$. **[4]**

(b) Find the perpendicular distance from the point with position vector $-\mathbf{i} - 2\mathbf{k}$ to Π_1. **[3]**

(c) The plane Π_2 has equation $3x + 2y - z = 14$. Find a vector equation of the line of intersection of Π_1 and Π_2. **[4]**

Solution 5

(a)

Mark scheme

- Normal $= (\mathbf{j} + 2\mathbf{k}) \times (\mathbf{i} + 3\mathbf{j} + \mathbf{k}) = (-5, 2, -1)$ (or $(5, -2, 1)$). Through $(-3, -1, -1)$: $5x - 2y + z = -14$.

Solution 5

(b)

Mark scheme

- $$\frac{|(-2,-1,1)\cdot(-5,2,-1)|}{\sqrt{5^2+2^2+1^2}} = \frac{7}{\sqrt{30}} \approx 1.28.$$

Solution 5

(c)

Mark scheme

- Common point e.g. $(0,7,0)$; direction $= (5,-2,1) \times (3,2,-1) = (0,8,16) \parallel (0,1,2)$. So $\mathbf{r} = \begin{pmatrix} 0 \\ 7 \\ 0 \end{pmatrix} + \lambda \begin{pmatrix} 0 \\ 1 \\ 2 \end{pmatrix}$.

Question 6

9231/12 N25 ▪ Q6 ▪ 14 marks

The plane Π has equation $x + 3y + 2z = 1$.

(a) Find the perpendicular distance from the origin O to the plane Π. **[2]**

(b) Relative to O, the points A, B, C have position vectors $-\mathbf{j} + 2\mathbf{k}$, $2\mathbf{i} - \mathbf{k}$ and $2\mathbf{i} - \mathbf{j} - \mathbf{k}$ respectively. Find the acute angle between the planes OAB and Π. **[4]**

(c) Find an equation for the common perpendicular to the lines OC and AB. **[8]**

Solution 6

(a)

Mark scheme

- Distance $= \dfrac{|-1|}{\sqrt{1^2 + 3^2 + 2^2}} = \dfrac{1}{\sqrt{14}}$.

Solution 6

(b)

Mark scheme

- Normal to OAB: $\overrightarrow{OA} \times \overrightarrow{OB} = (1, 4, 2)$; normal to Π: $(1, 3, 2)$.
  $\cos\theta = \dfrac{1 + 12 + 4}{\sqrt{21}\,\sqrt{14}} = \dfrac{17}{\sqrt{294}}$, so the acute angle is 7.5.

Solution 6

(c)

Mark scheme

- $\overrightarrow{OC} = (2, -1, -1)$, $\overrightarrow{AB} = (2, 1, -3)$; $\overrightarrow{OC} \times \overrightarrow{AB} = (4, 4, 4) \parallel (1, 1, 1)$. Requiring it to meet both lines gives $\mathbf{r} = \left(\frac{3}{2}, -\frac{1}{4}, -\frac{1}{4}\right) + \lambda(1, 1, 1)$.

Question 4

9231/14 N25 ▪ Q4 ▪ 10 marks

Relative to the origin O, the points A, B, C and D have position vectors $\overrightarrow{OA} = 3\mathbf{i} + 5\mathbf{j} + 5\mathbf{k}$, $\overrightarrow{OB} = 2\mathbf{i} + 2\mathbf{j} + 2\mathbf{k}$, $\overrightarrow{OC} = 2\mathbf{i} + \mathbf{j} - \mathbf{k}$ and $\overrightarrow{OD} = \mathbf{i} - \mathbf{j} + 3\mathbf{k}$.

(a) Find an equation of the plane through A, B and C, giving your answer in the form $ax + by + cz = e$. **[5]**

(b) Find the shortest distance between the lines AB and CD. **[5]**

Solution 4

(a)

Mark scheme

- $\overrightarrow{AB} = -\mathbf{i} - 3\mathbf{j} - 3\mathbf{k}$, $\overrightarrow{AC} = -\mathbf{i} - 4\mathbf{j} - 6\mathbf{k}$. Normal $= \overrightarrow{AB} \times \overrightarrow{AC} = (6, -3, 1)$. Through $A(3, 5, 5)$: $6(3) - 3(5) + 5 = 8$, so $6x - 3y + z = 8$.

Solution 4

(b)

Mark scheme

- $\overrightarrow{CD} = (1,-1,3) - (2,1,-1) = (-1,-2,4)$. Common perpendicular $\overrightarrow{AB} \times \overrightarrow{CD} = (-18,7,-1)$. Shortest distance $= \dfrac{\left|\overrightarrow{AC} \cdot (-18,7,-1)\right|}{\sqrt{18^2+7^2+1^2}} = \dfrac{4}{\sqrt{374}} = 0.207$.

Question 6

9231/11 J25 ▪ Q6 ▪ 16 marks

The points A, B, C have position vectors $\mathbf{i} - 2\mathbf{k}$, $\mathbf{i} + 2\mathbf{j} + 2\mathbf{k}$, $2\mathbf{i} - \mathbf{j} - \mathbf{k}$ respectively.

(a) Find the equation of the plane ABC, giving your answer in the form $ax + by + cz = d$. **[5]**

(b) A point D has position vector $\mathbf{i} + t\mathbf{k}$, where $t \neq -2$. Find the acute angle between the planes ABC and ABD. **[4]**

(c) Find the values of t such that the shortest distance between the lines AB and CD is $\sqrt{2}$. **[7]**

Solution 6

(a)

Mark scheme

- $\overrightarrow{AB} = 2\mathbf{j} + 4\mathbf{k}$, $\overrightarrow{AC} = \mathbf{i} - \mathbf{j} + \mathbf{k}$. Normal $= \overrightarrow{AB} \times \overrightarrow{AC} = (6, 4, -2) \parallel (3, 2, -1)$. Through $A(1, 0, -2)$: $3(1) + 2(0) - (-2) = 5$, so $3x + 2y - z = 5$.

Solution 6

(b)

Mark scheme

- $\overrightarrow{AD} = (0, 0, t+2)$, so normal to $ABD = \overrightarrow{AB} \times \overrightarrow{AD} = (2t+4, 0, 0) \parallel (1, 0, 0)$.
  Then $\cos\theta = \dfrac{(3, 2, -1)\cdot(1, 0, 0)}{\sqrt{14}\cdot 1} = \dfrac{3}{\sqrt{14}}$, giving $\theta = 36.7$ $(0.641$ rad$)$.

Solution 6

(c)

Mark scheme

- $\overrightarrow{CD} = (-1, 1, t+1)$. Common perpendicular $\overrightarrow{AB} \times \overrightarrow{CD} = (2t-2, -4, 2) \parallel (t-1, -2, 1)$. Shortest distance $= \dfrac{|\overrightarrow{AC} \cdot (t-1, -2, 1)|}{\sqrt{(t-1)^2+4+1}} = \dfrac{|t+2|}{\sqrt{t^2-2t+6}}$. Setting $= \sqrt{2}$: $(t+2)^2 = 2(t^2-2t+6) \Rightarrow t^2 - 8t + 8 = 0 \Rightarrow t = 2(2+\sqrt{2}) \approx 6.83$ or $t = 2(2-\sqrt{2}) \approx 1.17$.

Question 5

9231/13 J25 ▪ Q5 ▪ 11 marks

The plane Π has equation $\mathbf{r} = 2\mathbf{i} + 3\mathbf{j} - 2\mathbf{k} + \lambda(\mathbf{i} - 2\mathbf{j} - \mathbf{k}) + \mu(3\mathbf{i} + 2\mathbf{j} - 2\mathbf{k})$.

(a) Find a Cartesian equation of Π, giving your answer in the form $ax + by + cz = d$. **[4]**

(b) The point P has position vector $4\mathbf{i} + 2\mathbf{j} + 9\mathbf{k}$. Find the position vector of the foot of the perpendicular from P to Π. **[4]**

(c) The line l is parallel to the vector $3\mathbf{i} + 5\mathbf{j} - \mathbf{k}$. Find the acute angle between l and Π. **[3]**

Solution 5

(a)

Mark scheme

- Normal $= (1, -2, -1) \times (3, 2, -2) = (6, -1, 8)$. Through $(2, 3, -2)$: $6(2) - (3) + 8(-2) = -7$, so $6x - y + 8z = -7$.

Solution 5

(b)

Mark scheme

- Foot $\overrightarrow{OF} = \overrightarrow{OP} + t\,\mathbf{n} = (4+6t,\ 2-t,\ 9+8t)$. Substituting into Π: $6(4+6t)-(2-t)+8(9+8t) = -7 \Rightarrow 101t+94 = -7 \Rightarrow t = -1$. So $\overrightarrow{OF} = -2\mathbf{i}+3\mathbf{j}+\mathbf{k}$.

Solution 5

(c)

Mark scheme

- $\cos\alpha = \dfrac{(3,5,-1)\cdot(6,-1,8)}{\sqrt{35}\sqrt{101}} = \dfrac{5}{\sqrt{35}\sqrt{101}}$ (angle between *l* and the normal).
  The acute angle between *l* and Π is $90-\alpha = 4.8$.