Exercise walk-through

Summation of series ▪ 1.3

eXercise

You must be able to

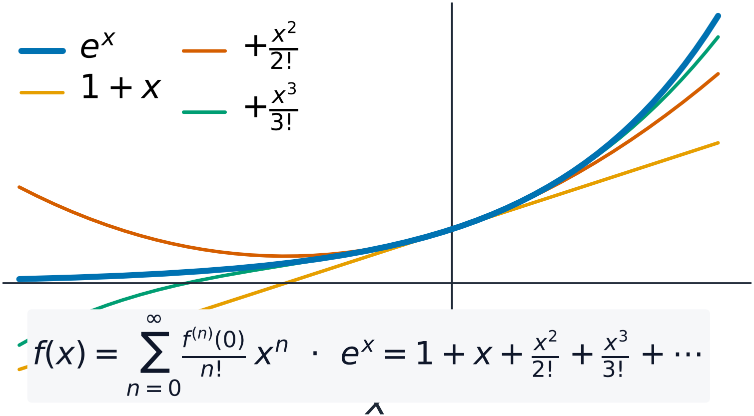
- use $\sum r = \frac{1}{2}n(n+1)$, $\sum r^2 = \frac{1}{6}n(n+1)(2n+1)$, $\sum r^3 = \frac{1}{4}n^2(n+1)^2$
- sum a series by the method of differences (telescoping)
- decide convergence and find a sum to infinity

Method — Standard results & differences

$$\sum_{r=1}^{n}(2r+1) = 2 \cdot \tfrac{1}{2}n(n+1) + n = n(n+2).$$

Differences: if $u_r = f(r) - f(r+1)$ then $\sum_{r=1}^{n} u_r = f(1) - f(n+1)$.

Method — Standard results & differences



$$f(x) = \sum_{n=0}^{\infty} \frac{f^{(n)}(0)}{n!} x^n \quad \cdot \quad e^x = 1 + x + \frac{x^2}{2!} + \frac{x^3}{3!} + \cdots$$

A Maclaurin-style series approximation building up term by term

Practice 2.1 ▪ 1 mark

Evaluate $\sum_{r=1}^{n} r^2$ as a formula in n.

Solution 2.1

Worked steps

$\frac{1}{6}n(n+1)(2n+1)$ **B1**.

Practice 2.2 ▪ 2 marks

Find $\sum_{r=1}^{10}(3r-1)$.

Solution 2.2

Worked steps

$3 \cdot \frac{1}{2}(10)(11) - 10 = 165 - 10 = 155$ **M1** **A1**.

Exam-style 3.1 ▪ 1 mark

$\sum_{r=1}^{n} r^3$ equals:

A $\frac{1}{4}n^2(n+1)^2$

B $\frac{1}{6}n(n+1)(2n+1)$

C $\frac{1}{2}n(n+1)$

D n^3

Solution 3.1

- **A** $\frac{1}{4}n^2(n+1)^2$ ✓
- **B** $\frac{1}{6}n(n+1)(2n+1)$
- **C** $\frac{1}{2}n(n+1)$
- **D** n^3

Worked steps

A. $\sum r^3 = \frac{1}{4}n^2(n+1)^2$.

Exam-style 3.2 ▪ 2 marks

(a) Express $\dfrac{1}{r(r+1)}$ in partial fractions.

(b) Hence find $\displaystyle\sum_{r=1}^{n} \frac{1}{r(r+1)}$ by the method of differences, and state its sum to infinity.

Solution 3.2

Method: Standard results & differences

Worked steps

(a) $\dfrac{1}{r(r+1)} = \dfrac{1}{r} - \dfrac{1}{r+1}$ **M1** **A1**.

(b) $\displaystyle\sum_{r=1}^{n}\left(\frac{1}{r} - \frac{1}{r+1}\right) = 1 - \frac{1}{n+1} = \frac{n}{n+1}$ **M1** **M1** **A1**; as $n \to \infty$ the sum $\to 1$ **A1**.

Exam-style 3.3 ▪ 4 marks

Find $\sum_{r=1}^{n}(r^2 + r)$, giving your answer as a single factorised expression.

Solution 3.3

Worked steps

$\sum(r^2 + r) = \frac{1}{6}n(n+1)(2n+1) + \frac{1}{2}n(n+1)$ **M1**; $= \frac{1}{6}n(n+1)[(2n+1)+3]$ **M1**;
$= \frac{1}{6}n(n+1)(2n+4) = \frac{1}{3}n(n+1)(n+2)$ **M1** **A1**.