

Exercise walk-through

Rational functions and graphs ■ 1.2

eXercise

You must be able to

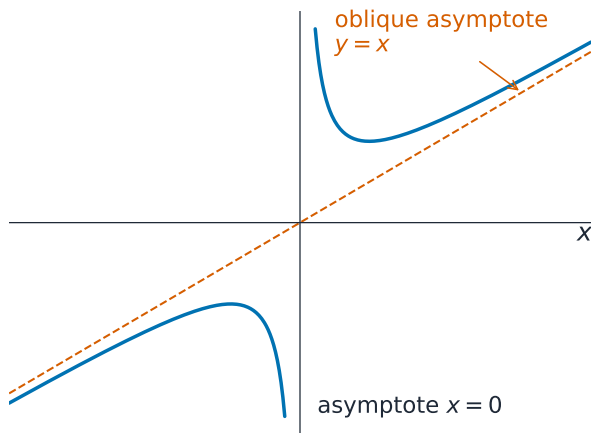
- find vertical and oblique asymptotes (divide out)
- find the set of values of a rational function using the discriminant
- relate $y = f(x)$ to $y = \frac{1}{f(x)}$, $y = |f(x)|$, $y^2 = f(x)$

Method — Asymptotes & range

$$y = \frac{x^2 + 1}{x} = x + \frac{1}{x}: \text{vertical asymptote } x = 0, \text{ oblique asymptote } y = x.$$

Range of $y = \frac{x^2 + 1}{x}$: $yx = x^2 + 1 \Rightarrow x^2 - yx + 1 = 0$; real x needs $y^2 - 4 \geq 0$, so $y \leq -2$ or $y \geq 2$.

Method — Asymptotes & range



A rational curve in two branches alongside a slant asymptote

Practice 2.1 ■ 1 mark

Write down the vertical asymptote of $y = \frac{2x + 1}{x - 3}$.

Solution 2.1

Method: Asymptotes & range

Worked steps

$$x = 3 \text{ (B1).}$$

Practice 2.2 ■ 2 marks

Find the oblique asymptote of $y = \frac{x^2}{x-1}$.

Solution 2.2

Method: Asymptotes & range

Worked steps

$$\frac{x^2}{x-1} = x + 1 + \frac{1}{x-1}, \text{ so oblique asymptote } y = x + 1 \quad \text{M1} \quad \text{A1}.$$

Exam-style 3.1 ■ 1 mark

The curve $y = \frac{x^2 - 4}{x}$ has an oblique asymptote:

A $y = x$

B $y = x - 4$

C $y = 2x$

D $y = -4$

Solution 3.1

Method: Asymptotes & range

A $y = x$



B $y = x - 4$

C $y = 2x$

D $y = -4$

Worked steps

A. $\frac{x^2 - 4}{x} = x - \frac{4}{x}$, so the oblique asymptote is $y = x$.

Exam-style 3.2 ■ 3 marks

A curve has equation $y = \frac{x^2 + 2}{x - 1}$.

- (a) Find the equations of the two asymptotes.
- (b) Use the discriminant to find the set of values that y **cannot** take.

Solution 3.2

Method: Asymptotes & range

Worked steps

(a) vertical $x = 1$ **B1**; divide: $\frac{x^2 + 2}{x - 1} = x + 1 + \frac{3}{x - 1}$, oblique $y = x + 1$ **M1** **A1**.

(b) $y(x - 1) = x^2 + 2 \Rightarrow x^2 - yx + (y + 2) = 0$ **M1**; real x needs $y^2 - 4(y + 2) \geq 0 \Rightarrow y^2 - 4y - 8 \geq 0$ **M1**; roots $y = 2 \pm 2\sqrt{3}$ **M1**; so y cannot take values in $2 - 2\sqrt{3} < y < 2 + 2\sqrt{3}$ **A1**.

Exam-style 3.3 ■ 4 marks

The function is $y = \frac{x}{x^2 + 1}$. Find the set of values y can take.

Solution 3.3

Worked steps

$$y(x^2 + 1) = x \Rightarrow yx^2 - x + y = 0 \quad \text{M1}; \text{ for real } x \text{ (with } y \neq 0), 1 - 4y^2 \geq 0 \quad \text{M1};$$
$$-\frac{1}{2} \leq y \leq \frac{1}{2} \quad \text{M1 A1}.$$