Exercise walk-through

Roots of polynomial equations ▪ 1.1

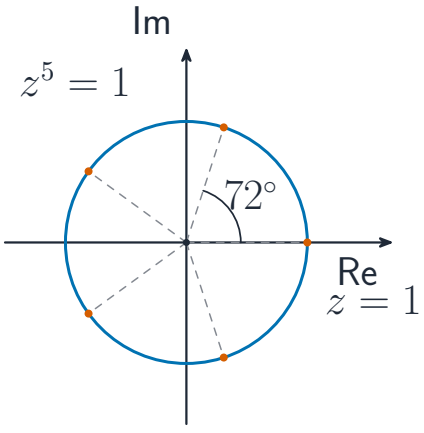
eXercise

You must be able to

- use $\sum \alpha$, $\sum \alpha\beta$, $\alpha\beta\gamma$ for quadratics and cubics
- evaluate symmetric functions such as $\sum \alpha^2$
- form an equation whose roots are a transformation of the originals

Method — Roots and coefficients

$x^2 - 5x + 6 = 0$ **has roots** α, β**. Find the equation with roots** $\alpha + 1, \beta + 1$**.**
$\alpha + \beta = 5$, $\alpha\beta = 6$; new sum $= 7$, new product $= \alpha\beta + \alpha + \beta + 1 = 12$. So $x^2 - 7x + 12 = 0$.
$\sum \alpha^2 = (\sum \alpha)^2 - 2\sum \alpha\beta$**.**



Practice 2.1 ▪ 2 marks

For $x^2 - 7x + 10 = 0$ with roots α, β, write down $\alpha + \beta$ and $\alpha\beta$.

Solution 2.1

Method: Roots and coefficients

Worked steps

$\alpha + \beta = 7$, $\alpha\beta = 10$ B1 B1.

Practice 2.2 ▪ 2 marks

Hence find $\alpha^2 + \beta^2$.

Solution 2.2

Method: Roots and coefficients

Worked steps

$\alpha^2 + \beta^2 = (\alpha + \beta)^2 - 2\alpha\beta = 49 - 20 = 29$ M1 A1.

Exam-style 3.1 ▪ 1 mark

For $2x^3 - 3x^2 + 4x - 5 = 0$ with roots α, β, γ, the value of $\alpha + \beta + \gamma$ is:

A $\frac{3}{2}$

B $-\frac{3}{2}$

C 2

D $\frac{5}{2}$

Solution 3.1

Method: Roots and coefficients

A $\frac{3}{2}$ ✓

B $-\frac{3}{2}$

C 2

D $\frac{5}{2}$

Worked steps

A. $\alpha+\beta+\gamma=-\dfrac{-3}{2}=\dfrac{3}{2}$.

Exam-style 3.2 ▪ 3 marks

The cubic $x^3 - 4x^2 + 5x - 2 = 0$ has roots α, β, γ.
(a) Write down $\sum \alpha$, $\sum \alpha\beta$ and $\alpha\beta\gamma$.
(b) Find $\alpha^2 + \beta^2 + \gamma^2$.

Solution 3.2

Method: Roots and coefficients

Worked steps

(a) $\sum \alpha = 4$, $\sum \alpha\beta = 5$, $\alpha\beta\gamma = 2$ B1 B1 B1.

(b) $\sum \alpha^2 = (\sum \alpha)^2 - 2\sum \alpha\beta = 16 - 10 = 6$ M1 M1 A1.

Exam-style 3.3 ▪ 4 marks

The equation $x^2 + 3x - 1 = 0$ has roots α, β. Find a quadratic equation with integer coefficients whose roots are $\dfrac{1}{\alpha}$ and $\dfrac{1}{\beta}$.

Solution 3.3

Method: Roots and coefficients

Worked steps

$\frac{1}{\alpha}+\frac{1}{\beta}=\frac{\alpha+\beta}{\alpha\beta}=\frac{-3}{-1}=3$ M1; $\frac{1}{\alpha\beta}=\frac{1}{-1}=-1$ M1; new equation $x^2-3x-1=0$ M1 A1.