

Learn these by heart before the exam.

Complex numbers

$$z = x + iy = r(\cos \theta + i \sin \theta) = re^{i\theta}$$

$$|z| = \sqrt{x^2 + y^2}, \quad \arg z = \arctan \frac{y}{x}$$

$$(\cos \theta + i \sin \theta)^n = \cos n\theta + i \sin n\theta \quad \text{de Moivre's theorem}$$

$$z = re^{i\theta} : z^{1/n} = r^{1/n} e^{i(\theta+2k\pi)/n} \quad n\text{-th roots are equally spaced around a circle}$$

Matrices

$$\det \begin{pmatrix} a & b \\ c & d \end{pmatrix} = ad - bc$$

$$M^{-1} = \frac{1}{\det M} \begin{pmatrix} d & -b \\ -c & a \end{pmatrix} \quad \text{singular (no inverse) when } \det M = 0$$

Eigenvalues and eigenvectors: $M\mathbf{v} = \lambda\mathbf{v}$; solve $\det(M - \lambda I) = 0$ for λ , then find each $\mathbf{v}$

$M = PDP^{-1}$, $M^n = PD^nP^{-1}$ diagonalisation; D holds the eigenvalues

$\det M$ is the area/volume scale factor of the transformation.

Series and induction

$$\sum_{r=1}^n r = \frac{1}{2}n(n+1)$$

$$\sum_{r=1}^n r^2 = \frac{1}{6}n(n+1)(2n+1)$$

$$\sum_{r=1}^n r^3 = \frac{1}{4}n^2(n+1)^2$$

Method of differences: write $u_r = f(r) - f(r+1)$ so the sum telescopes and most terms cancel

Proof by induction: prove for $n = 1$; assume true for $n = k$; prove it for $n = k + 1$; conclude true for all n

Calculus and polar

$$\text{Maclaurin: } f(x) = f(0) + f'(0)x + \frac{f''(0)}{2!}x^2 + \dots$$

$$\int \frac{1}{\sqrt{a^2 - x^2}} dx = \arcsin \frac{x}{a} + c$$

$$\int \frac{1}{a^2 + x^2} dx = \frac{1}{a} \arctan \frac{x}{a} + c$$

$$\text{Area (polar)} = \frac{1}{2} \int_{\alpha}^{\beta} r^2 d\theta$$

$$\cosh x = \frac{e^x + e^{-x}}{2}, \quad \sinh x = \frac{e^x - e^{-x}}{2}$$

$$\cosh^2 x - \sinh^2 x = 1$$

Differential equations

$\frac{dy}{dx} + P(x)y = Q(x)$: IF = $e^{\int P dx}$ first-order linear; multiply through by the integrating factor

Second-order (constant coefficients): solve the auxiliary equation $am^2 + bm + c = 0$; the general solution = complementary function + particular integral

Real distinct roots $\Rightarrow Ae^{m_1x} + Be^{m_2x}$; complex roots $\Rightarrow e^{px}(A \cos qx + B \sin qx)$.

Further mechanics and statistics

$$\text{Momentum } p = mv, \quad \text{impulse} = \Delta p$$

$$e = \frac{\text{speed of separation}}{\text{speed of approach}} \quad \text{Newton's law of restitution; } 0 \leq e \leq 1$$

$$E_{\text{elastic}} = \frac{\lambda x^2}{2l} \quad \text{energy stored in an elastic string/spring}$$

$$\chi^2 = \sum \frac{(O - E)^2}{E} \quad \text{goodness of fit; compare with the critical value}$$

Type I and Type II error: Type I: reject a true H_0 ; Type II: accept a false H_0