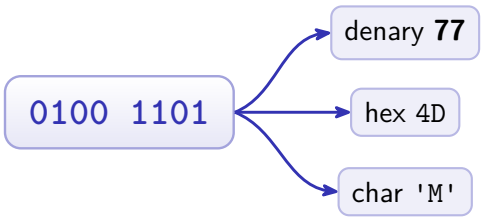
Information Representation

AS Computer Science ▪ Topic 1

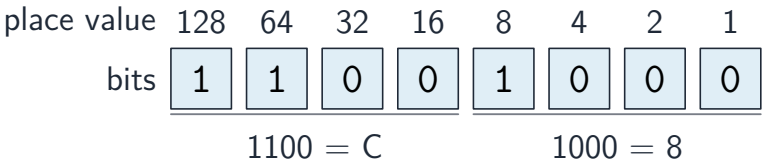
A-Level Computer Science



What we will cover

1. Number systems
2. Binary arithmetic
3. BCD & hexadecimal
4. Character codes
5. Images
6. Sound
7. Compression

1 Number systems



$$200 = 128 + 64 + 8 = \texttt{11001000}_2 = \texttt{C8}_{16}$$

Three number systems

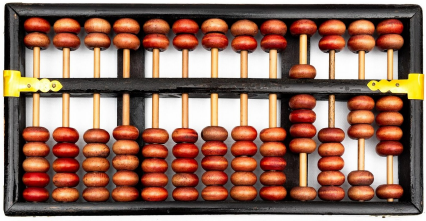
Three ways to write the **same number** – here, thirteen:

denary
十进制 ▪ base 10
digits 0–9

13

binary
二进制 ▪ base 2
0 and 1

1101

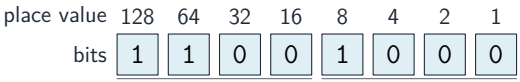


An abacus represents numbers by place value – the same idea behind decimal, binary and hexadecim

a **byte** 字节 = 8 **bits** 位; one hex digit = 4 bits

Converting between bases

- **denary** → **binary**: subtract the largest **place value** 位值
- **binary** → **hex**: group into **nibbles** 半字节
- **hex** → **denary**: digit × place value



$$200 = 128 + 64 + 8 = 11001000_2 = C8_{16}$$

Convert denary 200

$200 = 128 + 64 + 8 \Rightarrow$ 1100 1000; 1100=C, 1000=8 $\Rightarrow$ hex C8.

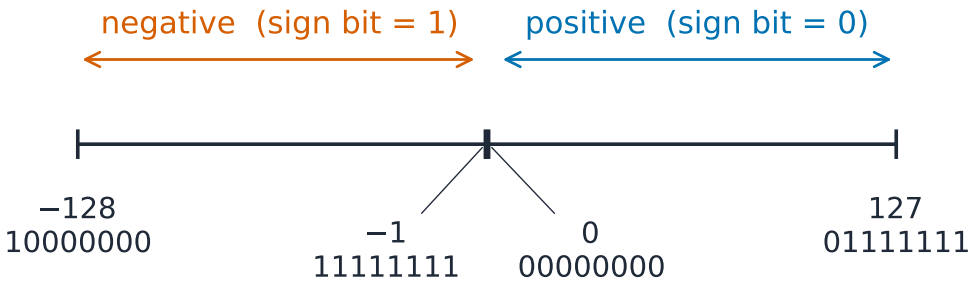
Binary vs decimal prefixes

Two families look alike – powers of 10 against powers of 2. A 1 TB drive holds 10^{12} bytes; an OS reporting in TiB shows a smaller number for the same drive.

Decimal	Binary
kilo $= 10^3$	kibi $= 2^{10} = 1024$
mega $= 10^6$	mebi $= 2^{20}$
giga $= 10^9$	gibi $= 2^{30}$

same name, different size

2 Binary arithmetic



8-bit two's complement: −128…127 · MSB = sign bit

Signed and unsigned integers

A bit pattern carries no sign of its own – the same byte is two numbers, read as **unsigned** 无符号 or as **signed** 有符号.

8 bits	**as unsigned** every bit a place value	**as signed** top bit is the sign
00000000	0	0
01111111	127	$+127$
10000000	128	-128
11111111	255	-1
range for n bits	$0 \ldots 2^n - 1$	$-2^{n-1} \ldots 2^{n-1} - 1$

Addition & overflow

Add column by column from the right, carrying as in denary.

> **Overflow** 溢出: the result needs more bits than the **register** 寄存器 can hold.

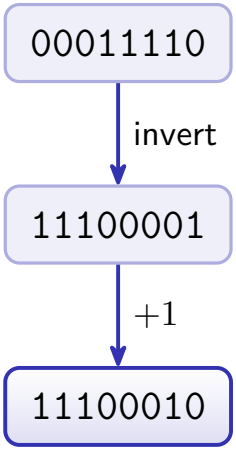
$$\begin{array}{rr} & 1111 \\ + & 0001 \\ \hline \text{overflow bit} \rightarrow & 10000 \end{array}$$

Subtraction by two's complement

For $A - B$: form the **two's complement** 补码 of B (invert, add 1), add, drop the final carry.

$100 - 30$ (8-bit)

00011110→11100010;
01100100+11100010
→ 01000110= 70. ✓

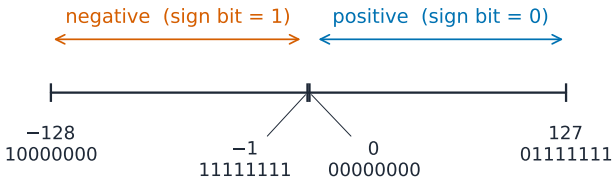


Two's complement signed integers

- **sign bit** 符号位 = **most significant bit** 最高有效位 (1 = negative)
- read it: invert, $+1$, then negate
- range: -2^{n-1} to $2^{n-1}-1$

10110100

invert$+1$ $=$01001100$=$ $76 \Rightarrow -76$



8-bit two's complement: −128…127 · MSB = sign bit

One's complement – the older scheme

One's complement 反码: negate by only inverting (no $+1$).

- $+30 = 00011110$, $-30 = 11100001$
- drawback: two zeros

> Two's complement has one zero and shares a circuit for $+$ and $-$.

$$00000000 = +0$$

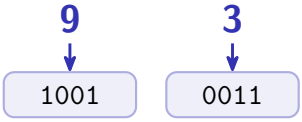
$$11111111 = -0$$

two zeros – wasteful

3 BCD & hexadecimal

Binary Coded Decimal

In **BCD** 二进码十进数, each denary digit gets its own 4 bits.



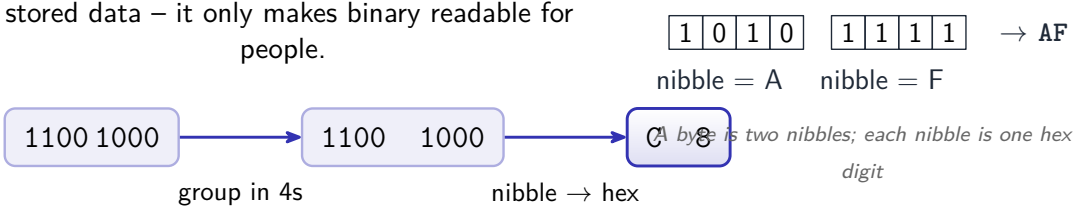
In pure binary the same number is 0101 1101 – a different pattern of the same 8 bits.

- only 0000–1001 are valid; 1010–1111 unused
- each digit drives a display directly, and money avoids binary rounding

*Drives a **7-segment display** 七段显示器*

Hexadecimal in practice

One hex digit = 4 bits, so one byte is exactly two hex digits – used for **memory addresses** 内存地址 (`0x7FFE`), colour codes and MAC addresses. Hex does not change the stored data – it only makes binary readable for people.



memory address 内存地址 `0x7FFE` ▪ colour `#FF8800` ▪ MAC `AC:DE:48:00:11:22`

4 Character codes

character	code (denary)	binary
A	65	01000001
a	97	01100001
0	48	00110000
(space)	32	00100000

ASCII

Text = numbers – each character has a **code point** 码点 in a **character set** 字符集.

ASCII: 7 bits, 128 code points ▪ **Extended ASCII**: 8 bits, 256

character	code (denary)	binary
A	65	01000001
a	97	01100001
0	48	00110000
(space)	32	00100000

Unicode

A universal character set – almost every script.

- **encodings** 编码: **UTF-8** (1–4 bytes), UTF-16, UTF-32
- portable, multilingual in one file
- larger files for English-only text

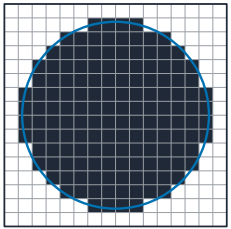
A ⟶ U+0041

中 ⟶ U+4E2D

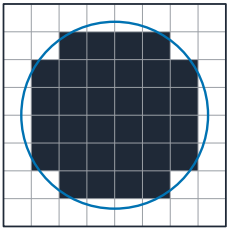
é ⟶ U+00E9

one code point each

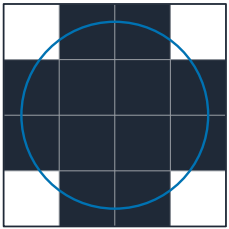
5 Images



A
16×16 pixels (high)



B
8×8 pixels



C
4×4 pixels (low)

fewer, larger pixels $\Rightarrow$ lower resolution, less detail

Bitmap images

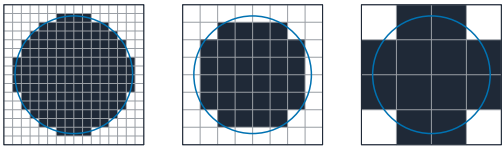
A **bitmap** 位图 stores every **pixel** 像素 in a grid, behind a **file header** 文件头 that records the resolution and colour depth.

- **resolution** 图像分辨率: w × h
- **colour depth** 颜色深度: bits/pixel

size in bits = w × h × depth

3000 × 2000 at 24 bpp

1.44×10^8 bits $\div 8 \div 1024^2 \approx 17\,\mathrm{MiB}$

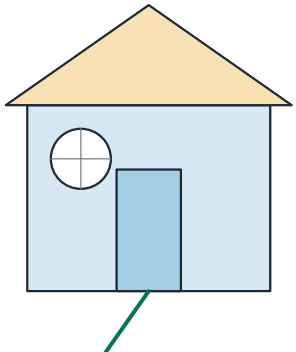


Changing settings

- **lower image resolution** 图像分辨率 → smaller file, less detail; blocky when enlarged (**screen resolution** 屏幕分辨率 is a different thing – the monitor's own pixel grid).
- **lower colour depth** → smaller file, but smooth shades show banding.
- **higher** of either → larger file, better quality.

Vector graphics

A **vector graphic** 矢量图形 stores the instructions to draw the image – a **drawing list** 绘图列表 of **drawing objects** 绘图对象, each a geometric **primitive** 图元 with its own properties. Redrawn crisply at any size.



the file is a list of shapes:

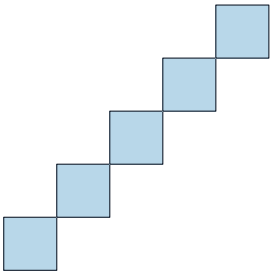
triangle — roof	3 points
circle — window	$(0.75, 1.85)$, $r\,0.42$
rectangle — body	$(0, 0)$, 3.4×2.6
rectangle — door	$(1.25, 0)$, 0.9×1.7
line — path	$(1.7, 0) \to (1.1, -0.85)$

Bitmap vs vector

Use	Better
Photograph	**bitmap** 位图
Painting, texture	bitmap
Logo, icon	vector
Engineering dwg	vector

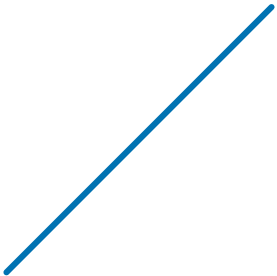
Bitmap wins on smooth tonal detail per area; vector scales without blur.

bitmap, enlarged



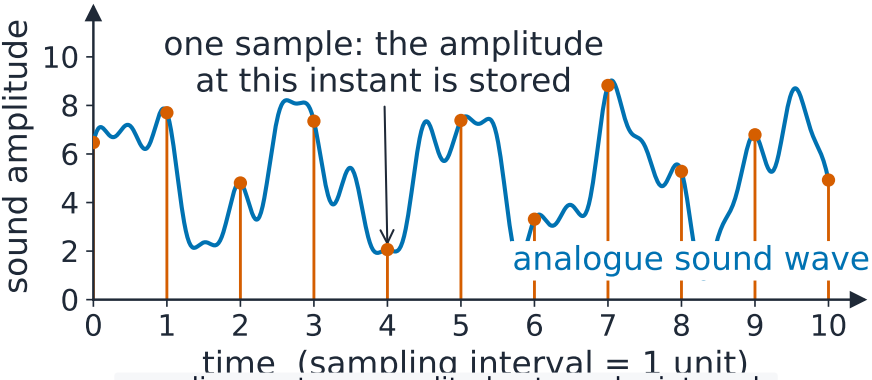
edges look jagged

vector, enlarged



edges stay smooth

6 Sound



sampling captures amplitude at regular intervals

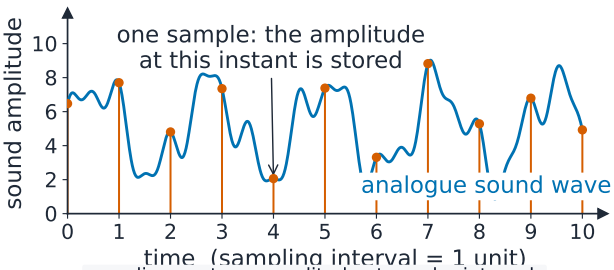
Sampling sound

Analogue data 模拟数据 → **digital data** 数字数据 by **sampling** 采样:

- **sampling rate** 采样率 (CD $44.1\,\mathrm{kHz}$)
- **sampling resolution** 采样分辨率 – bits per **amplitude** 振幅, set by **quantisation** 量化

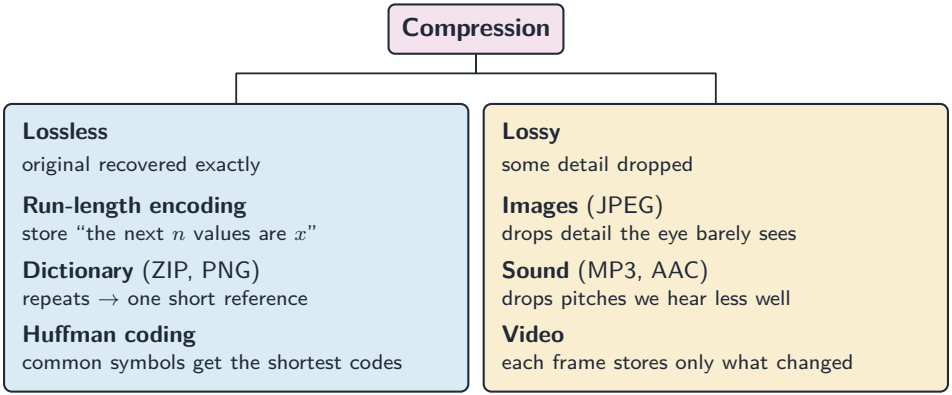
10 s stereo CD

$44100 \times 16 \times 10 \times 2 \approx 1.68\,\mathrm{MiB}$



sampling captures amplitude at regular intervals

7 Compression

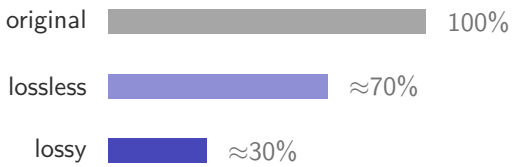


Lossless vs lossy

lossless 无损 – recovers data exactly (ZIP, PNG).

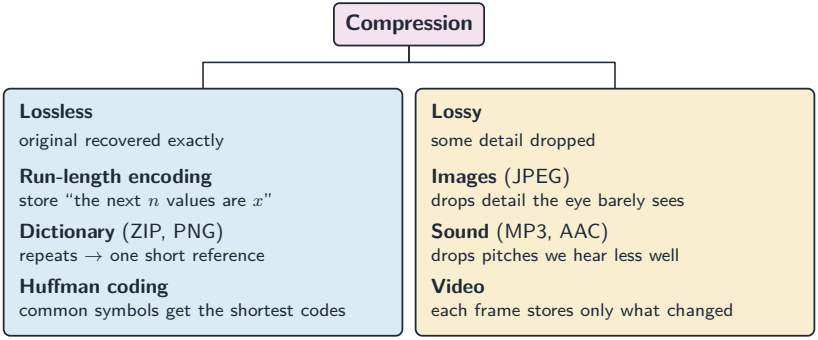
lossy 有损 – drops detail for smaller files (JPEG, MP3).

> **Compression** 压缩 saves storage and **bandwidth** 带宽. Video does both at once: **spatial** 空间 within each frame, **temporal** 时间 between them – most frames store only what *changed*.



same picture, three file sizes

Compression methods

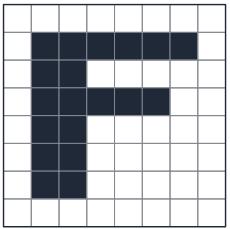


lossless 无损 recovers the original exactly – use it where every bit must survive (documents, code, medical images). **lossy** 有损 drops detail for much smaller files (JPEG, MP3, streaming video).

Run-length encoding

Store each **run** 行程 of identical values once, as a **(count, value)** pair – **lossless** 无损, so the original comes back exactly. Best on flat areas; useless on noisy data, where a run of one costs more than the value it replaces.

8×8 bitmap

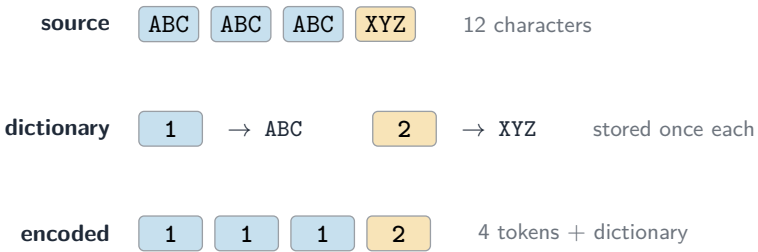


binary	**RLE code**
00000000	8W
01111110	1W 6B 1W
01100000	1W 2B 5W
01111100	1W 5B 2W
01100000	1W 2B 5W
01100000	1W 2B 5W
01100000	1W 2B 5W
00000000	8W

Key: 1 = black (B), 0 = white (W). RLE stores *count* + *colour*: e.g. 01111110 → 1W 6B 1W

Dictionary coding

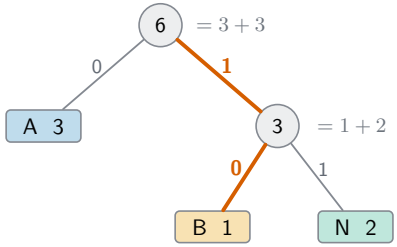
Dictionary methods 字典压缩 store each repeated **sequence** 序列 once in a **dictionary**; every occurrence then becomes its short **index** 索引. **Lossless** 无损, and best where whole sequences repeat – text, source code, flat colour in a PNG.



The decoder rebuilds the dictionary as it reads, so the original returns exactly — **lossless** (ZIP, PNG).

Huffman coding

Give the commonest symbol the shortest code – a **variable-length code** 变长编码, still **lossless** 无损. No code starts another, so the decoder needs no separator, and the average code length approaches the data's **entropy** 信息熵.



each circle joins the two smallest counts

Walk **down from the root**, writing a bit at each branch: left $= 0$, right $= 1$.

A $= 0$

B $= 10$ right, then left

N $= 11$

BANANA becomes 10 0 11 0 11 0 — **10 bits**, against 12 for fixed 2-bit codes: the commonest letter is nearest the root.

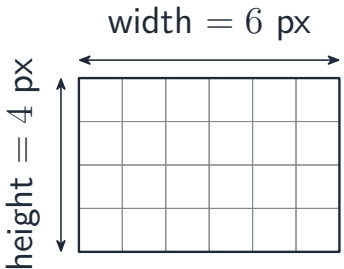
Exam tips

- Show working for base conversions: denary $\rightarrow$ binary by place values, binary $\rightarrow$ hexadecimal in **nibbles** (groups of 4 bits).
- For **two's complement** the MSB is negative; to negate, **invert and add 1**; watch for **overflow** when the sign bit flips wrongly.
- Distinguish **bitmap** (pixels; file size = width $\times$ height $\times$ colour depth) from **vector** (drawing commands; scales without loss).
- Sound file size depends on **sample rate** $\times$ **bit depth** $\times$ **time** – more of each means better quality but a bigger file.
- Compare **lossless vs lossy** compression and give a use for each.

8 Recap

Topic 1 – key takeaways

1. denary / binary / hex; one hex digit = 4 bits
2. two's complement for signed integers
3. text = code points; images bitmap/vector; sound sampled
4. compression: lossless or lossy



pixels $= 6 \times 4 = 24$

size $= 24 \times \mathbf{8} = \mathbf{192}$ bits

($\mathbf{8}$ = colour depth, bits per pixel)

Check yourself

1. Convert denary 173 to binary and hex.
2. 8-bit two's-complement 11111011 $= ?$
3. 1920×1080 at 24 bpp – size in MiB?
4. Why is JPEG lossy but PNG lossless?

16 pixels
$\Rightarrow$ **6W 4B 6W**
(3 runs, not 16)

Answers: `10101101/AD` ▪ -5 ▪ $\approx 5.9\,\mathrm{MiB}$ ▪ JPEG drops unseen detail; PNG keeps every pixel.