Computational thinking and Problem-solving

A-Level Computer Science

Searching algorithms

A **search** finds a **target value** in a collection (often an **array** 数组) and returns its position, or "not found".

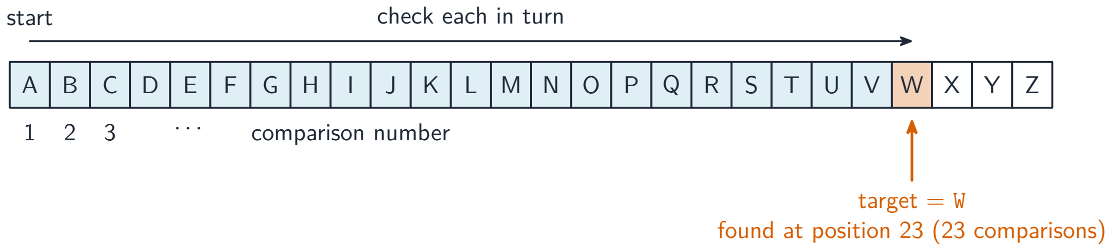
Searching a sorted list, like a phone book, is far faster than checking every entry one by one

Linear search

A **linear search** 线性查找 walks from start to end, comparing each element with the target:

```
FOR i ← 1 TO n
    IF A[i] = target THEN RETURN i
NEXT i
RETURN -1   // not found
```

No preparation is needed, so it works on any list. Worst case $O(n)$ (target at the end or absent); best case 1 comparison. Use it on **unsorted** data or small lists. (The returned `-1` is a **sentinel** value — an impossible position that means "not found"; the caller tests `IF result = -1`.)



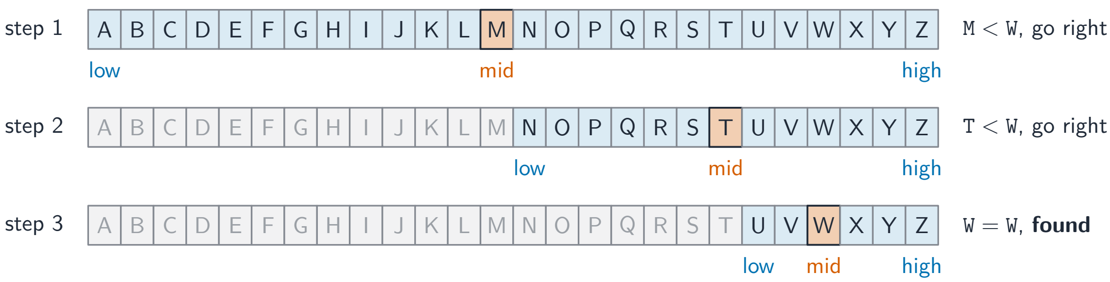
Linear search checks every letter in turn — 23 comparisons to find W

Binary search

A **binary search** 二分查找 needs the data **sorted**. Look at the middle element; if it is the target, done; if the target is smaller, search the left half, else the right half — halving the range each time:

```
low ← 1
high ← n
WHILE low <= high DO
    mid ← (low + high) DIV 2
    IF A[mid] = target THEN RETURN mid
    IF A[mid] < target THEN
        low ← mid + 1
    ELSE
        high ← mid - 1
    ENDIF
ENDWHILE
RETURN -1
```

Worst case $O(\log_2 n)$ — for a million items, about 20 comparisons. Much faster than linear search on large sorted arrays, but you must sort first (a one-off $O(n \log n)$ cost), worth it if you search many times.



Binary search halves the range each step (low / mid / high) — just 3 comparisons to find W

Sorting algorithms

Bubble sort

A **bubble sort** 冒泡排序 repeatedly walks the array, swapping adjacent pairs that are out of order, so the largest "bubbles" to the end each pass:

```
FOR pass ← 1 TO n - 1
    swapped ← FALSE
    FOR i ← 1 TO n - pass
        IF A[i] > A[i + 1] THEN
            temp ← A[i]
            A[i] ← A[i + 1]
            A[i + 1] ← temp
            swapped ← TRUE
        ENDIF
    NEXT i
    IF swapped = FALSE THEN EXIT FOR   // already sorted
NEXT pass
```

Best case $\mathrm{O}(n)$ (already sorted, with the early exit); average/worst $\mathrm{O}(n^2)$. Simple but slow for large n.

Insertion sort

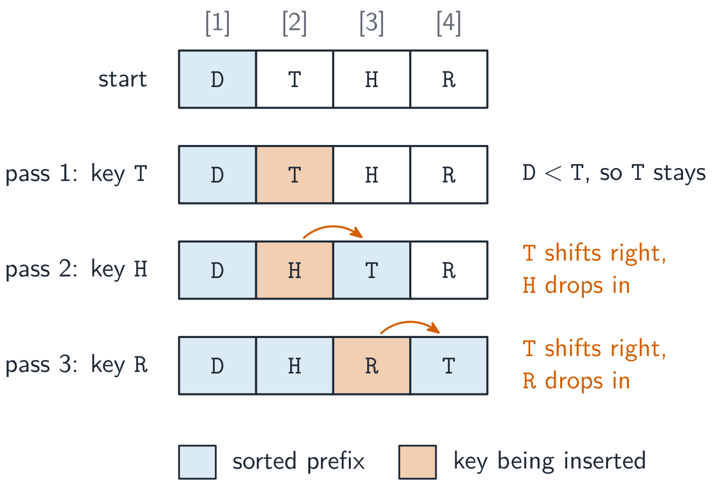
An **insertion sort** 插入排序 builds a sorted prefix from the left, inserting each new element into place by shifting larger ones right:

```
FOR i ← 2 TO n
    key ← A[i]
    j ← i - 1
    WHILE j >= 1 AND A[j] > key DO
        A[j + 1] ← A[j]
        j ← j - 1
    ENDWHILE
    A[j + 1] ← key
NEXT i
```

Best case $\mathrm{O}(n)$ (already sorted); worst $\mathrm{O}(n^2)$. Good for **small** or **nearly-sorted** arrays. It sorts **in place** 原地 and is **stable** 稳定 (keeps the order of equal elements).

Tracing a sort

A common task is to show the array after each outer pass. For `[D, T, H, R]` with insertion sort: pass 1 (key T) no change; pass 2 (key H) → `[D, H, T, R]`; pass 3 (key R) → `[D, H, R, T]`.

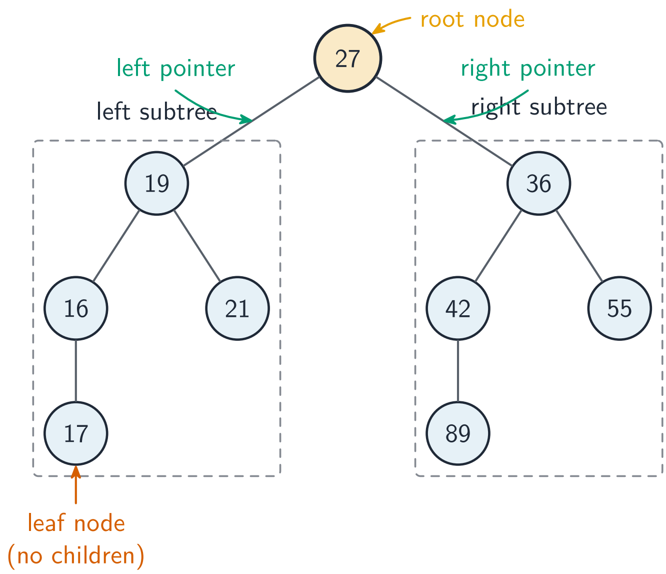


An insertion sort of `[D, T, H, R]`, shifting each key into its place pass by pass

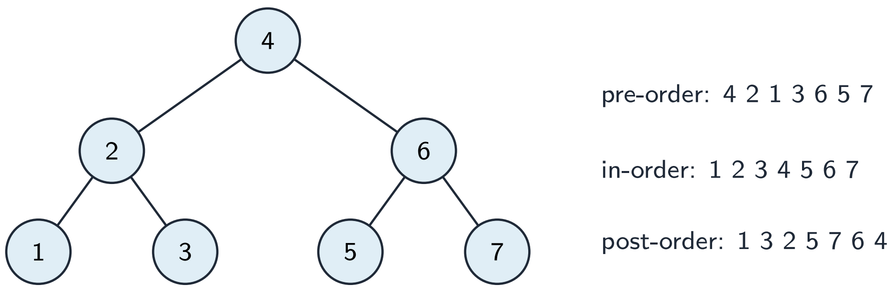
ADTs in algorithms

The **Abstract Data Types** (ADTs) from Topic 10 appear inside many algorithms: a **stack** 栈 drives depth-first traversal and undo; a **queue** 队列 drives breadth-first traversal and print ordering; a **linked list** 链表 lets data grow and shrink.

ADTs can be built from other ADTs, not just from arrays: a queue from **two stacks**; a stack from a linked list (push = prepend a head **node** 节点); a queue from a linked list with head and tail **pointers** 指针; a **binary tree** 二叉树 from nodes with two child pointers; a **dictionary** 字典 stores key→value pairs (often on a hash table). Layering this way separates concerns — the algorithm using the ADT need not know how it is built.



A binary tree: each node has up to two child nodes



Three depth-first traversals of a binary tree: pre-order, in-order (sorted order) and post-order

Comparing algorithms

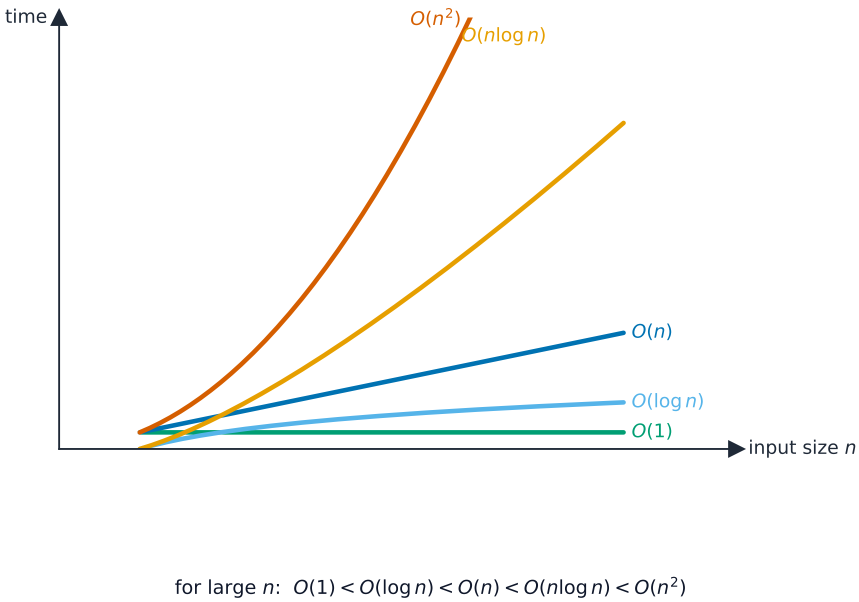
Time complexity

Time complexity 时间复杂度 is how the running time grows with input size n, written in **Big-O notation** 大 O 表示法 (the dominant term): $\mathrm{O}(1)$ constant, $\mathrm{O}(\log n)$ binary search, $\mathrm{O}(n)$ linear search, $\mathrm{O}(n \log n)$ good sorts, $\mathrm{O}(n^2)$ bubble/insertion sort. A smaller order is better at scale, even if another algorithm is faster for small n.

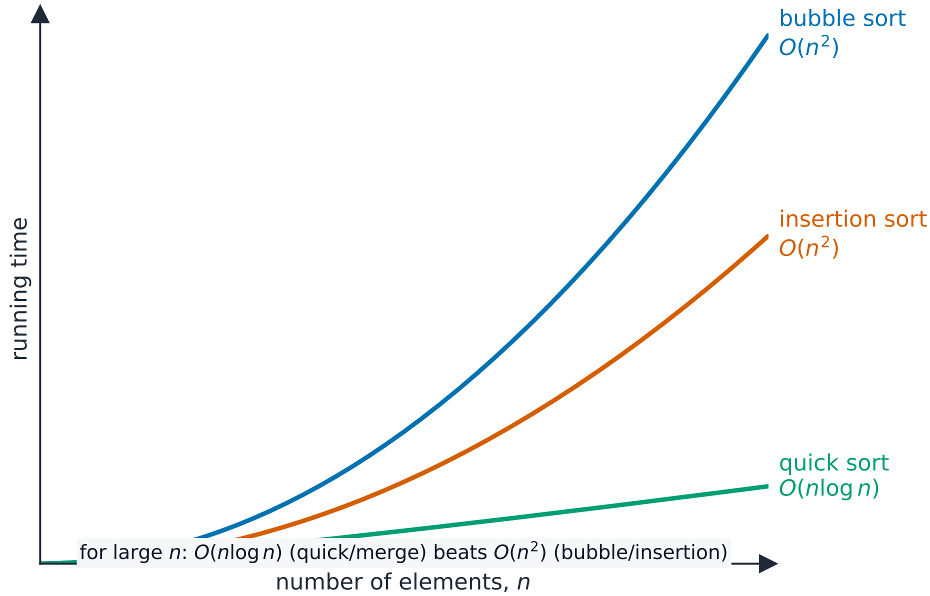
To make that concrete: to sort a million items, an $O(n \log n)$ sort finishes in a fraction of a second, while an $O(n^2)$ sort can take minutes.

Worked example. A sorted list holds 1000 items. How many comparisons does each search need in the worst case?

A linear search checks items one at a time, so it may need up to 1000 comparisons — this is $O(n)$. A binary search halves the list each step, so it needs at most $\lceil \log_2 1000 \rceil = 10$ comparisons — this is $O(\log n)$. Doubling the list to 2000 items adds only **one** comparison to the binary search, but up to another 1000 to the linear search — which is why the order of growth, not raw speed, decides the winner at scale.



How the common orders of growth compare: a smaller order wins at scale



How sorting time grows with the number of elements n: $O(n^2)$ sorts climb away from an $O(n \log n)$ sort

Space complexity

Space complexity 空间复杂度 is the extra memory needed. Bubble and insertion sort use O(1) extra (in place); merge sort uses O(n); recursion uses stack memory proportional to its depth. There is often a time–memory trade-off.

Other criteria

Simplicity (easier to code and maintain), stability, and adaptiveness (faster on nearly-sorted data). The right algorithm depends on the data and the constraints.

Recursion

Recursive algorithms use **recursion** 递归: the routine calls itself with a smaller version of the same problem, until a **base case** 基本情形 ends the chain. It has two parts: the base case (small enough to solve directly — without it the recursion never stops) and the **recursive case** 递归情形 (reduce the input and call itself).

Factorial 阶乘:

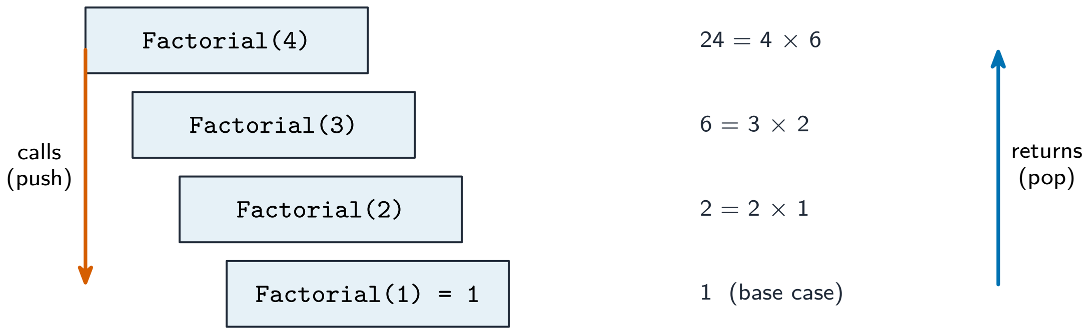
```
FUNCTION Factorial(n : INTEGER) RETURNS INTEGER
    IF n = 0 OR n = 1 THEN
        RETURN 1
```

```
    ELSE
        RETURN n * Factorial(n - 1)
    ENDIF
ENDFUNCTION
```

Recursion is natural for self-similar problems: trees, **divide-and-conquer** 分治 (binary search, merge sort), and nested data. When it is a poor fit, a loop is usually cleaner.

Tracing a recursive call

For `Factorial(4)`: the calls go down to `Factorial(1)=1`, then **unwinding** multiplies back up: `2*1=2`, `3*2=6`, `4*6=24`. Final result `24`. Track each pending call on a stack.



Recursion uses the call stack: calls push frames down to the base case, then returns unwind back up

Risks

- **infinite recursion** if the base case is missed — crashes with a **stack overflow** 栈溢出.
- high memory use for deep recursion.
- slow if it repeats work (naive Fibonacci is exponential — use a loop or **memoisation** 记忆化).

What the compiler does for recursive code

Recursion needs **each call to have its own copy** of its **parameters** 参数 and **local variables** 局部变量. The compiler keeps these on the **call stack** 调用栈. For each call it pushes a **stack frame** 栈帧 holding the parameters, the local variables, and the **return address** 返回地址 (where to resume in the caller). When the function returns, the return value is handed back, the frame is popped, and control resumes at the return address.

Because each call has its own frame, recursive calls don't trample each other's variables. The stack can grow large for deep recursion, which is why very deep recursion may overflow it. This is the same call-and-return mechanism used for ordinary (non-recursive) calls — there is no special "recursion mechanism".

Exam tips

- Match each algorithm to its **Big-O**: linear search $O(n)$, binary search $O(\log n)$, bubble/insertion $O(n^2)$, good sorts $O(n \log n)$.
- **Binary search needs a sorted list** and halves the search space each step.
- A **recursive** routine needs a base case and a call to itself; explain how deep recursion overflows the call stack.
- Trace a sort or search with a table when asked, showing each pass.