Exercise walk-through

Data Representation ▪ 1.1

eXercise

You must be able to

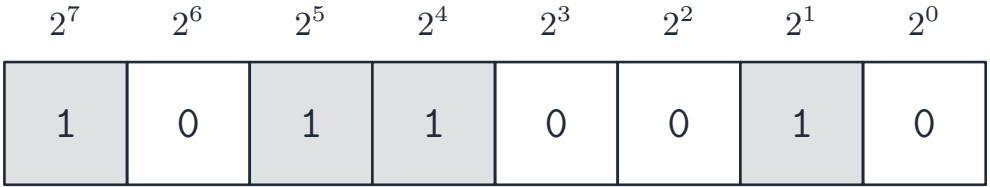
- convert between **binary 二进制**, **denary 十进制** and **hexadecimal 十六进制**, and use **BCD 二进码十进数**
- state the **minimum number of bits** needed to store a value
- add 8-bit binary numbers and **name and describe overflow 溢出**
- use **two's complement 补码**: write a negative denary number in binary, read a two's-complement value, and subtract
- represent character data with **ASCII** and **Unicode**, and explain why Unicode is used

Method — Binary ↔ denary

Multiply each bit by **2 raised to its position** (positions count from 0 at the right), then add. This same method works for *any* base — for hexadecimal you use powers of 16.

To go **denary → binary**, divide by 2 again and again; the **remainders 余数**, read from **bottom to top**, are the bits.

Method — Binary ↔ denary



$$(1\times2^7)+(1\times2^5)+(1\times2^4)+(1\times2^1)$$
$$=\ 128+32+16+2=\ \mathbf{178}$$

Method — Binary ↔ denary

$178 \div 2 = 89$	remainder **0**
$89 \div 2 = 44$	remainder **1**
$44 \div 2 = 22$	remainder **0**
$22 \div 2 = 11$	remainder **0**
$11 \div 2 = 5$	remainder **1**
$5 \div 2 = 2$	remainder **1**
$2 \div 2 = 1$	remainder **0**
$1 \div 2 = 0$	remainder **1**

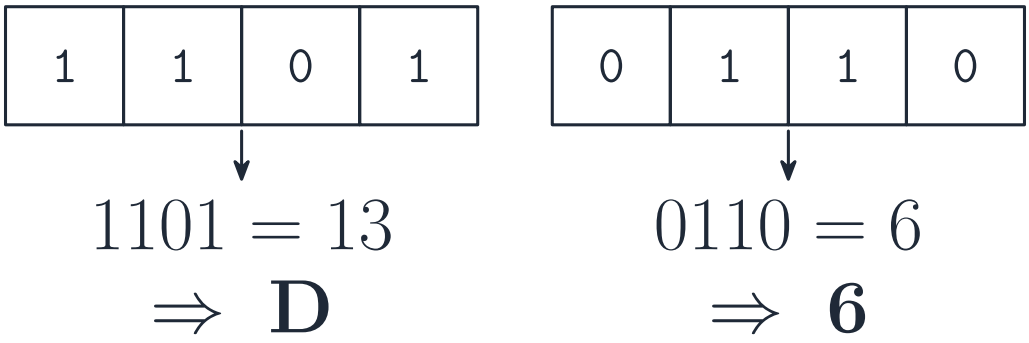
read up

$= 10110010$

Method — Hexadecimal

Each hex digit is exactly 4 bits (a **nibble 半字节**): 0–9, then A = 10 up to F = 15. **Hex → denary** uses the powers method with base 16: 2F $= (2 \times 16^1) + (15 \times 16^0) = 32 + 15 = \mathbf{47}$. **Shortcut binary ↔ hex:** group the bits into nibbles of 4 from the right and convert each.

Method — Hexadecimal



gives D6

Method — Number of bits to store a value

To store the values 0 to N, you need the smallest number of bits b with $2^b > N$. Example: to store 0–200, $2^7 = 128$ is too small but $2^8 = 256 > 200$, so **8 bits**. (An 8-bit byte holds 0–255.)

Method — Binary addition and overflow

Add one column at a time from the right; when a column makes 2, write 0 and **carry** a 1.

```
   0101 1100   (92)
 + 0011 0110   (54)
 ------------
   1001 0010   (146)
```

Method — Binary addition and overflow

Overflow is when the answer is too big for the register (over 255 for 8 bits): a 1 carries out of the leftmost (bit-7) column, so the stored 8-bit result is wrong.

```
   1100 1000   (200)
+  0101 0000   (80)
------------
1  0001 1000   (carry out of bit 7 → overflow)
```

Method — Binary addition and overflow

In words: **overflow** is when a calculation gives a result with more bits than the register can hold, so the carry out of the most-significant bit is lost and the stored answer is incorrect.

Method — Two's complement: negative numbers

In two's complement the **most significant bit** is the **sign bit** (0 = positive, 1 = negative). For 8 bits the range is -128 to $+127$.

- **Write** -18**:** take $+18 = 00010010$, **invert** every bit $\rightarrow 11101101$, **add 1** $\rightarrow 11101110$.
- **Read** 11101110**:** the sign bit is 1, so it is negative. Invert $\rightarrow 00010001$, add 1 $\rightarrow 00010010 = 18$, so the value is -18.
- **Subtract** by adding a negative. $45 - 18$: add 00101101 (45) and 11101110 (-18), then **discard** the carry out of bit 7:

```
  0010 1101   (45)
+ 1110 1110   (-18)
------------
1 0001 1011   (discard carry → 27)
```

Method — Binary Coded Decimal (BCD)

BCD stores each denary digit on its own as a 4-bit code — *not* the plain binary of the whole number. For 59: $5 \rightarrow$ 0101 and $9 \rightarrow$ 1001, so 59 is 0101 1001. Each nibble uses only 0–9; patterns 1010–1111 are invalid.

Method — Character sets: ASCII and Unicode

A computer stores text as numbers: each character has a **code point 码点** fixed by a character set.

- **ASCII** uses **7 bits** → 128 characters (basic English letters, digits, punctuation, control codes). Example: `'A'` = 65 = `1000001`.
- **Unicode** is a universal set covering (almost) every script plus symbols and emoji, so it needs **more bits per character**.

A character's stored code is just a binary number, so you convert it like any other: the value `0010 0111` $= 32 + 4 + 2 + 1 = 39$.

Why Unicode? It represents **far more characters** than ASCII (every language, not just English), at the cost of **larger files** for plain English text.

Practice 2.1 ▪ 1 mark

Convert binary 01101010 to denary.

Solution 2.1

Method: Hexadecimal

Worked steps

$01101010 = 64 + 32 + 8 + 2 = 106$ A1.

Practice 2.2 ▪ 2 marks

Convert denary 200 to 8-bit binary.

Solution 2.2

Method: Hexadecimal

Worked steps

Repeated division by 2 gives remainders, read bottom-up $\rightarrow$ 11001000 M1 A1 *method*.

Practice 2.3 ▪ 2 marks

Convert binary 11110001 to (a) denary and (b) hexadecimal.

Solution 2.3

Method: Hexadecimal

Worked steps

(a) $\texttt{11110001} = 128 + 64 + 32 + 16 + 1 = 241$ **A1**.

(b) split into `1111` | `0001` = `F1` **A1**.

Practice 2.4 ▪ 2 marks

State the **minimum number of bits** needed to store the denary value 500. Show how you decided.

Solution 2.4

Worked steps

$2^8 = 256$ is too small for 500, but $2^9 = 512 > 500$, so **9 bits** M1 A1 *compares powers of 2.*

Practice 2.5 ▪ 2 marks

Represent the denary number 59 in **BCD**.

Solution 2.5

Worked steps

$5 \to 0101$, $9 \to 1001$, so $0101 \;\; 1001$ M1 A1.

Practice 2.6 ▪ 2 marks

Add the 8-bit numbers 01011100 and 00110110. State whether overflow occurs.

Solution 2.6

Worked steps

01011100 (92) + 00110110 (54) $= 146 =$ 10010010; $146 \leq 255$, so **no overflow** M1 A1 *addition*.

Exam-style 3.1 ▪ 4 marks

Add the 8-bit binary numbers `10110100` and `01101101`. Show your working, then **name and describe** the error that occurs.

3.2 (a) Convert the hexadecimal number `3D` to denary. Show your working.

(b) Convert the denary number 158 to hexadecimal. Show your working.

3.3 (a) Write the denary number -45 as an 8-bit two's complement binary number. Show your working.

(b) The 8-bit two's complement number `11101110` is stored. Calculate the denary value it represents.

Solution 3.1

Worked steps

Adding the columns: \

```
   1011 0100   (180)
+  0110 1101   (109)
------------
   0010 0001   (carry out of bit 7)
```

Solution 3.1

$180 + 109 = 289 > 255$. The error is **overflow** M1 A1 *addition*, *result '00100001'*: the result needs 9 bits but only 8 are stored, so the carry out of the most-significant bit is lost and the stored answer is wrong B1 B1 *name*, *description*.

Exam-style 3.4 ▪ 2 marks

A computer stores text using a character set.
(a) The text is stored using the **Unicode** character set instead of **ASCII**. Give **two** advantages of using Unicode.
(b) A character has the binary code `0010 0111`. Convert this code to denary.

Solution 3.4

Method: Character sets: ASCII and Unicode

Worked steps

(a) Any **two**: Unicode can represent characters from (almost) every language / many more characters than ASCII; it includes symbols and emoji; it avoids the regional clashes of extended ASCII B1 B1.

(b) 0010 0111 $= 32 + 4 + 2 + 1 = 39$ A1.

Exam-style 3.5 ▪ 1 mark

A program stores currency amounts using **BCD** rather than ordinary binary.
(a) State what **BCD** stands for.
(b) Give one **advantage** of using BCD for this purpose.

Solution 3.5

Method: Character sets: ASCII and Unicode

Worked steps

(a) Binary Coded Decimal B1.
(b) Each denary digit is stored exactly, so values like money are not changed by rounding errors when converting fractions to binary (or: BCD digits map directly to a display) B1.