Data representation

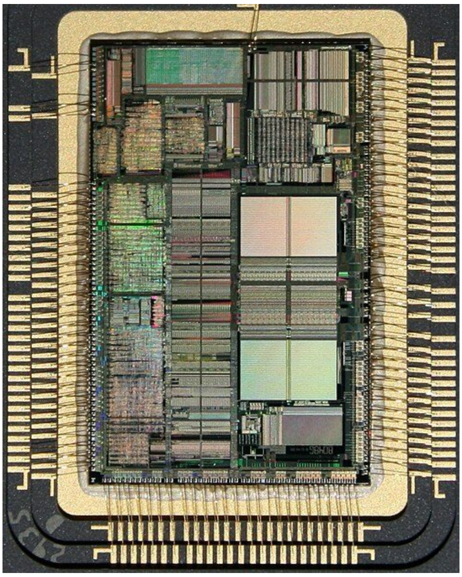
IGCSE Computer Science

Why computers use binary

A computer can only work with two states: on and off. You write these as 1 and 0. A system that uses only two digits 数字 is called **binary** 二进制 (base 2).

Computers represent all data — numbers, text, sound and images — as binary strings of 0s and 1s

Every kind of data 数据 — numbers, text, sound and images — must be changed into binary before a computer can use it. The computer processes this binary using **logic gates** 逻辑门, and stores it in **registers** 寄存器 (small, fast stores inside the processor 处理器).



A microprocessor holds millions of tiny transistors, each a switch that is on (1) or off (0) — the physical basis of binary

Number systems

A **number system** 数制 is a way of writing numbers using a fixed set of digits. You need three of them.

System	Base	Digits used
Denary	10	0–9
Binary	2	0 and 1
Hexadecimal	16	0–9 then A–F

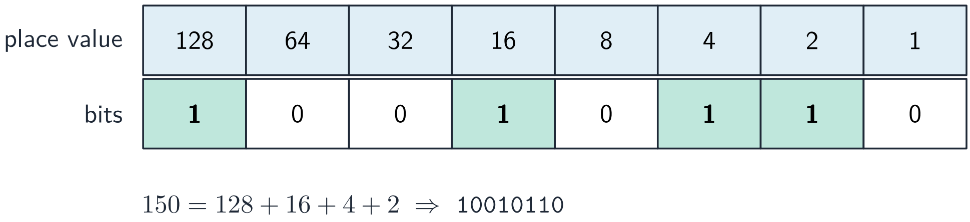
- **denary** 十进制 is the normal counting system (also called decimal).
- **binary** uses only 0 and 1.
- **hexadecimal** 十六进制 (hex) uses sixteen digits: 0–9, then A, B, C, D, E, F stand for 10, 11, 12, 13, 14, 15.

The **base** 基数 tells you how many different digits a system uses.

Place value

Each column in a number has a **place value** 位值. In binary the place values double from right to left. For an 8-bit number they are:

```
128  64  32  16  8  4  2  1
```



An 8-bit place-value chart: the 1s sit under the values that add up to 150

One **bit** 位 is a single 0 or 1. Eight bits make one **byte** 字节. Four bits (half a byte) is a **nibble** 半字节.

Converting between number systems

Denary → binary. Write the place values. Put a 1 under each value you need so they add up to your number; put 0 under the rest.

Example: change denary 150 to binary. $150 = 128 + 16 + 4 + 2$.

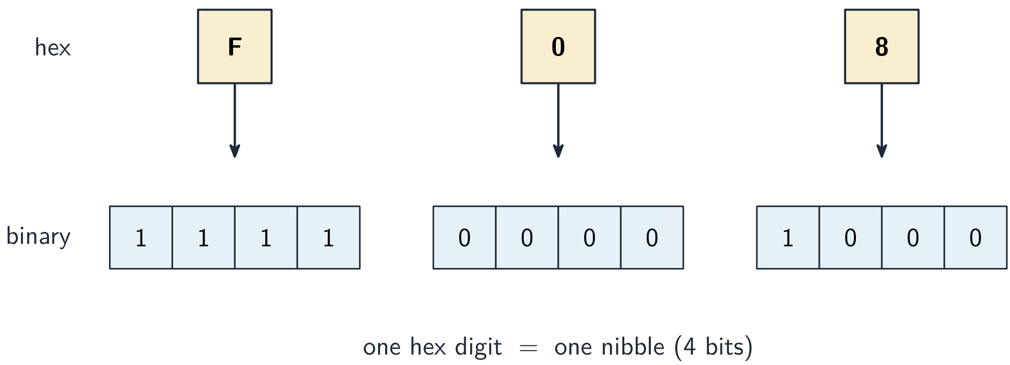
```
128 64 32 16 8 4 2 1
  1  0  0  1 0 1 1 0
```

So $150 =$ `10010110`.

Binary → denary. Add the place values where there is a 1. `10010110` $= 128 + 16 + 4 + 2 = 150$.

Hexadecimal → binary. Change each hex digit into its own 4-bit group (a nibble).

Example: hex F08. $F = 1111$, $0 = 0000$, $8 = 1000$, so F08 = `1111 0000 1000`.



Each hex digit maps to its own 4-bit nibble — F08 = 1111 0000 1000

Binary → hexadecimal. Group the bits into nibbles of 4, starting from the right. Change each nibble to one hex digit.

Denary → hexadecimal. The easy way is to change to binary first, then binary to hex.

This table helps with the hex letters:

Denary	Binary	Hex
10	1010	A
11	1011	B
12	1100	C
13	1101	D
14	1110	E
15	1111	F

Cambridge questions use binary numbers up to 16 bits long.

Worked example. Convert denary 100 to 8-bit binary, then to hexadecimal.

$100 = 64 + 32 + 4$, so the binary is `01100100`. Splitting into nibbles, `0110 0100` = 6 and 4, so the hexadecimal is `64`.

Why hexadecimal is used

Hex is shorter than binary and easier for people to read and write. One hex digit replaces 4 binary digits, so you make fewer mistakes. The value does not change — hex is just a shorter way to show the same binary.

Computer scientists use hex for:

- MAC addresses and IPv6 addresses
- colour codes in HTML (for example `#FF0000` is red)
- **memory addresses 内存地址** and error codes
- showing the contents of memory (a "memory dump")

Binary addition

You can add two 8-bit binary numbers, column by column from the right, just like denary. The rules for one column are:

A	B	Result bit	Carry
0	0	0	0
0	1	1	0
1	0	1	0
1	1	0	1

When a carry also comes in, $1 + 1 + 1 = 1$ with a carry of 1.

Example: add `01110110` (118) and `00110000` (48).

```
  0 1 1 1 0 1 1 0    (118)
+ 0 0 1 1 0 0 0 0    (48)
-------------------
  1 0 1 0 0 1 1 0    (166)
```

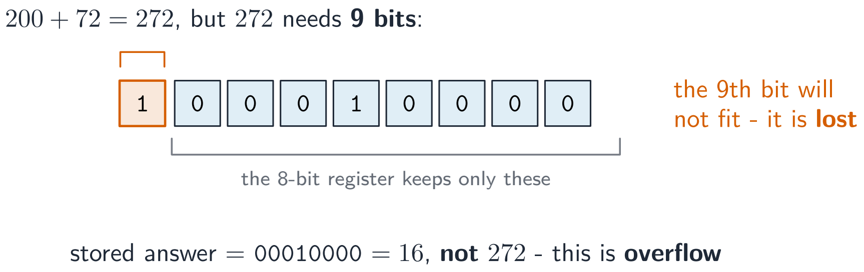
carries	1	1	1						
	0	1	1	1	0	1	1	0	118
+	0	0	1	1	0	0	0	0	48
	1	0	1	0	0	1	1	0	166

Adding column by column; the carries ripple to the left. 118 + 48 = 166

Overflow

An 8-bit register can hold denary values from 0 to 255 only. If an addition gives a result above 255, the answer needs a 9th bit. The register cannot hold this extra bit, so it is lost. This is called **overflow** 溢出 (an overflow error). It happens when a value goes outside the limit the register can store.

Example: `11001000` (200) + `01001000` (72) = 272. In binary that is `1 00010000`, which needs 9 bits. The leading 1 will not fit in 8 bits, so the stored answer is wrong.



Adding 200 and 72 needs 9 bits, but an 8-bit register drops the ninth, so the answer is wrong

Logical binary shift

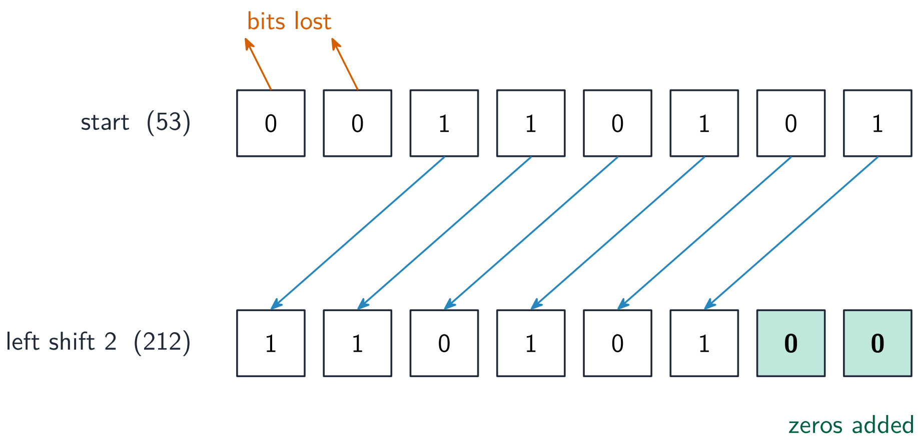
A **logical binary shift** 逻辑二进制移位 moves all the bits left or right by a number of places.

- Bits that move off the end of the register are **lost**.
- **Zeros** are added at the empty end.

A **left shift** multiplies the number by 2 for each place moved. A **right shift** divides it by 2 for each place; the right-most bits (the **least significant bit(s)** 最低有效位) are lost.

Example: left shift `00110101` (53) by 2 places.

```
start:          0 0 1 1 0 1 0 1
left shift 2:   1 1 0 1 0 1 0 0
```



A left shift of 2: every bit moves 2 places left, the top bits are lost and zeros fill the right

The result is `11010100` (212), which is 53×4. The two left-most bits were lost and two zeros came in on the right. If a 1 is pushed off the end, that information is gone for good.

Two's complement

So far the numbers were positive. **Two's complement** 补码 lets an 8-bit register hold negative numbers too.

In two's complement, the left-most bit (the **most significant bit** 最高有效位, or MSB) has a *negative* place value:

```
-128  64  32  16  8  4  2  1
```

- If the MSB is 0, the number is positive.
- If the MSB is 1, the number is negative.

To make a positive number negative: write the positive binary, flip every bit (0↔1), then add 1.

Example: make −40.

- +40 = `00101000`
- flip the bits = `11010111`
- add 1 = `11011000`

So −40 = `11011000`. Check by adding the place values: $-128 + 64 + 16 + 8 = -40$.

sign bit (negative)

place value	−128	64	32	16	8	4	2	1
bits	**1**	**1**	0	**1**	**1**	0	0	0

$-128 + 64 + 16 + 8 = -40$

The most significant bit is worth −128, so 11011000 = −128 + 64 + 16 + 8 = −40

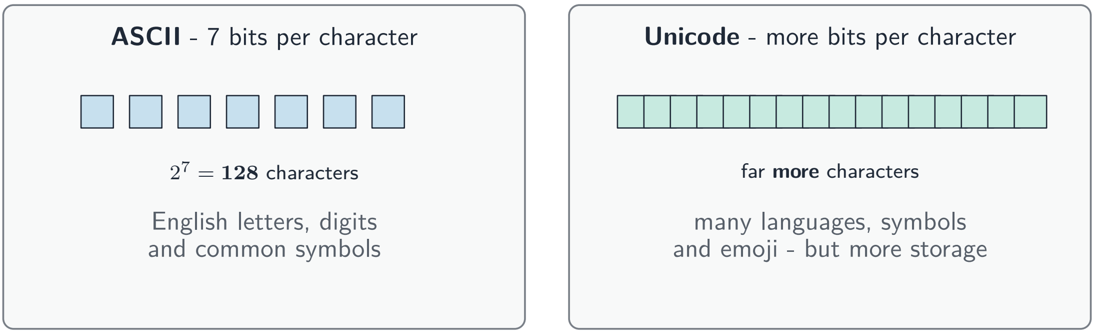
To read a negative two's complement number, just add the place values (the MSB counts as −128). The range of an 8-bit two's complement number is −128 to +127.

Representing text

Computers store text by giving every character a number, then storing that number in binary. The set of characters a computer can use, together with their numbers, is a **character set** 字符集.

- **ASCII** uses 7 bits per character, so it has 128 different characters. This is enough for English letters, digits and common symbols.
- **Unicode** uses more bits per character. It can represent far more characters — many languages, plus symbols and **emoji** 表情符号.

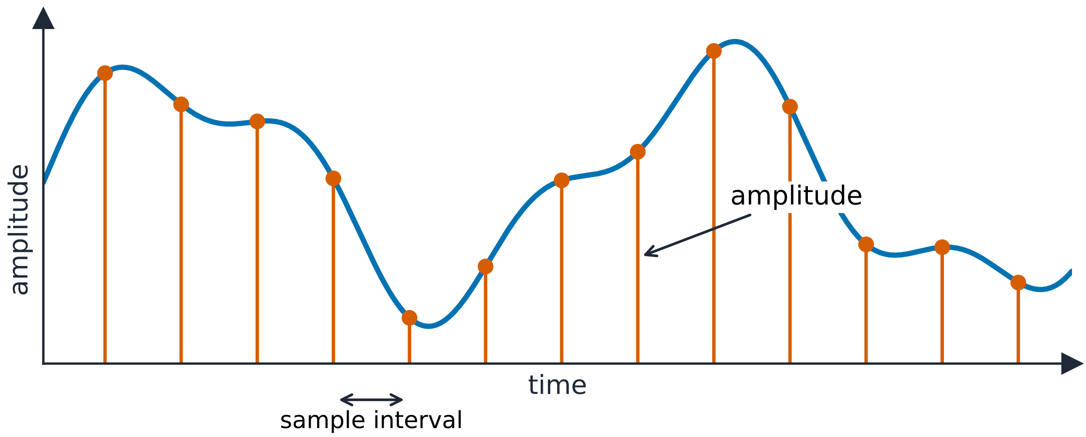
Because Unicode has more characters, it needs more bits per character than ASCII, so the same text takes more storage 存储.



ASCII uses 7 bits for 128 characters; Unicode uses more bits for far more characters but more storage

Representing sound

A **sound wave** 声波 is smooth and always changing. To store it, the computer measures the height of the wave at regular moments. This is called **sampling** 采样, and each measurement is a sample.



sample rate = samples/s · Nyquist $\geq 2f_{max}$

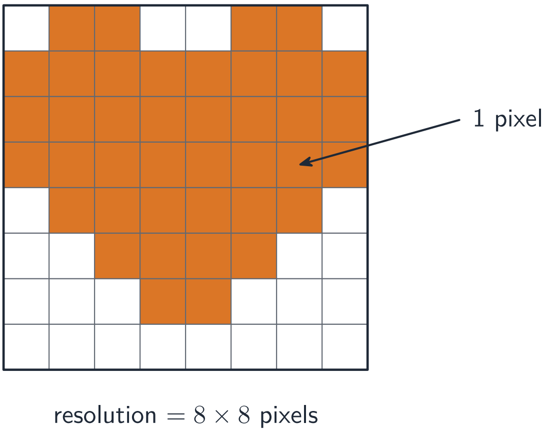
Sampling records the wave's height (amplitude) at regular moments

- **sample rate** 采样率 is the number of samples taken each second (measured in Hz).
- **sample resolution** 采样分辨率 is the number of bits used for each sample. The height of the wave at a sample point is its **amplitude** 振幅.

A higher sample rate and a higher sample resolution give a more accurate recording, but a larger file.

Representing images

A computer image is made of a grid of small dots called **pixels** 像素.



A bitmap is a grid of pixels; the resolution is how many pixels it has

- **resolution** 分辨率 is the number of pixels in the image (for example 1920×1080).
- **colour depth** 颜色深度 is the number of bits used to store the colour of each pixel.

A higher resolution and a higher colour depth give a better-quality image, but a larger file.

Measuring data storage

Data storage is measured in the units below. A nibble is 4 bits and a byte is 8 bits; from the kibibyte upward, each unit is 1024 times the one before it (because $1024 = 2^{10}$, which fits binary).

Unit	**Equals**
bit	a single 0 or 1
nibble	4 bits
byte	8 bits
kibibyte (KiB)	1024 bytes
mebibyte (MiB)	1024 KiB
gibibyte (GiB)	1024 MiB
tebibyte (TiB)	1024 GiB
pebibyte (PiB)	1024 TiB
exbibyte (EiB)	1024 PiB

Hard-disk platters: storage is measured in bytes — knowing file size needs width × height × colour depth for images

Calculating file size

Image file size (in bits) = resolution × colour depth = width × height × colour depth.

Example: an image is 1024 × 1024 pixels with a colour depth of 2 bytes (= 16 bits).

- bits = $1024 \times 1024 \times 16 = 16\,777\,216$ bits
- bytes = $\div 8 = 2\,097\,152$ bytes
- KiB = $\div 1024 = 2048$ KiB
- MiB = $\div 1024 = 2$ MiB

Sound file size (in bits) = sample rate × sample resolution × length in seconds.

Always divide by 1024 (not 1000) to change to KiB, MiB and so on. Give your answer in the unit the question asks for.

Worked example. A sound is recorded for 30 seconds at a sample rate of 8,000 Hz with a sample resolution of 16 bits. Find the file size in kibibytes (KiB).

- bits = $8\,000 \times 16 \times 30 = 3\,840\,000$ bits
- bytes = $3\,840\,000 \div 8 = 480\,000$ bytes
- KiB = $480\,000 \div 1024 \approx 469$ KiB

Compression

Compression 压缩 makes a file smaller. A smaller file:

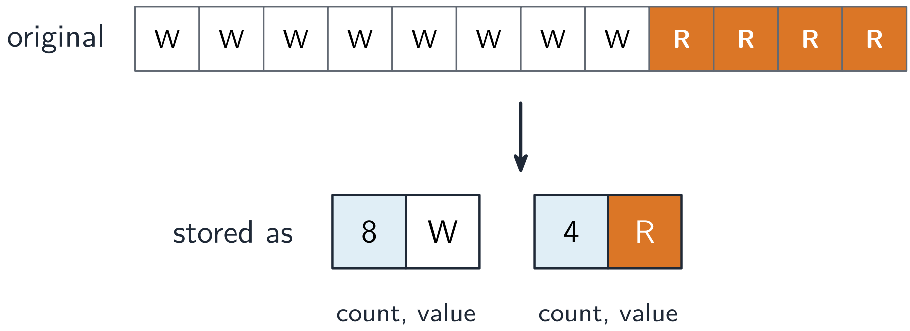
- uses less storage space,
- needs less **bandwidth** 带宽 (the amount of data a connection can carry),
- takes a shorter time to send (a shorter **transmission** 传输 time).

There are two types.

Lossless compression

Lossless 无损 compression makes the file smaller with **no permanent loss** of data. The original file can be rebuilt exactly.

One method is **run-length encoding** 行程编码 (RLE). It replaces a run of repeated values with one copy of the value and a count of how many times it repeats. For example `WWWWWWWW` (8 whites) is stored as ”8 W”. This works well when data has many repeats.



Run-length encoding stores each run once as a count and a value

Lossy compression

Lossy 有损 compression makes the file much smaller by **permanently removing** some data. The removed data cannot be got back. For example:

- reducing the resolution or colour depth of an image,
- reducing the sample rate or sample resolution of a sound.

Use lossless when you must keep every detail (text and program files). Use lossy for photos, music and video, where a small loss of quality is worth a much smaller file.

Exam tips

- Convert denary → binary by subtracting the place values (128, 64, 32 ...); binary → denary by adding the place values that hold a 1.
- To convert to hex, group the binary into **nibbles** of 4 bits from the right; each nibble is exactly one hex digit.
- **Overflow** happens when a result needs more bits than the register has (an 8-bit register only holds 0–255), so the extra bit is lost.

- File size in bits: for an image, width × height × colour depth; for sound, sample rate × resolution × seconds. Divide by 8 for bytes, then by 1024 for each larger unit.
- **Lossless** compression keeps every bit (text; run-length encoding); **lossy** permanently removes data (photos, music) for a much smaller file.