# Electromagnetic Induction

AP Physics C: Electricity and Magnetism

## Magnetic Flux

**Magnetic flux** 磁通量 measures how much magnetic field passes through a surface. For a uniform field through a flat loop it is the dot product $\Phi_B = \vec{B} \cdot \vec{A} = BA\cos\theta$; in general it is the **surface integral** 面积分

$$\Phi_B = \int \vec{B} \cdot d\vec{A}.$$

The **area vector** 面积矢量 is perpendicular to the surface (outward from a closed one), and the sign of the flux comes from the dot product. Flux changes if $B$ changes, the area changes, or the loop turns – keep all three routes in mind, because each one is an exam question.

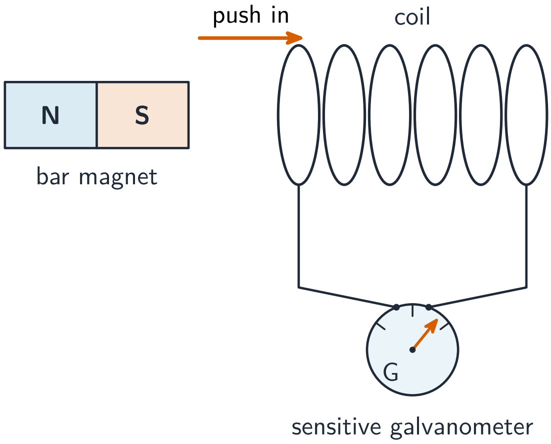
## Electromagnetic Induction

A **changing** flux induces an **emf** 电动势 – **Faraday's law** 法拉第定律:

$$\varepsilon = -\frac{d\Phi_B}{dt}.$$

With constant area, $\varepsilon = -A\,\dfrac{dB_\perp}{dt}$; with constant field, $\varepsilon = -B\,\dfrac{dA_\perp}{dt}$. A coil of $N$ turns multiplies the single-loop emf: $|\varepsilon_{\text{sol}}| = N\left|\dfrac{d\Phi_B}{dt}\right|$.

**Lenz's law** 楞次定律 is the minus sign: the **induced current** 感应电流 flows so that its own magnetic field *opposes the change* in flux that created it. Push a magnet toward a loop and the loop pushes back; pull it away and the loop pulls it in. Use the **right-hand rule** 右手定则 to turn "oppose the change" into a current direction.

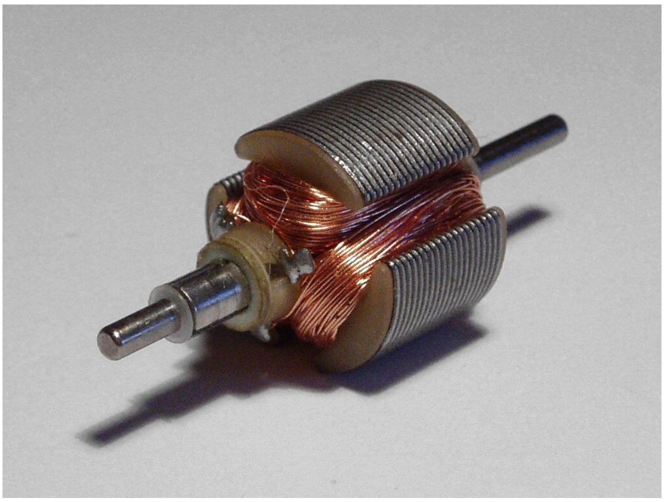



*Moving a magnet into a coil induces an e.m.f. that drives a current*

A rod of length $L$ sliding at speed $v$ across a field is the special case worth memorising – the **motional emf 动生电动势** $\varepsilon = BLv$.

**Worked example.** A 0.20 m rod slides at 3.0 m/s across a 0.50 T field: $\varepsilon = BLv = 0.50(0.20)(3.0) = 0.30$ V. Equivalently, if a single loop's flux drops from 0.020 Wb to 0.008 Wb in 0.030 s, the average emf is $\varepsilon = \dfrac{0.012}{0.030} = 0.40$ V.

Faraday's law is also the third of **Maxwell's equations 麦克斯韦方程组**, in a deeper form: a changing magnetic flux creates a circulating *electric field*, $\oint \vec{E} \cdot d\vec{l} = -\dfrac{d\Phi_B}{dt}$ – that field is what pushes the charges around the loop. Together, Maxwell's equations predict **electromagnetic waves 电磁波** travelling at $c = 1/\sqrt{\varepsilon_0 \mu_0}$ (you should know this connection, but AP will not ask you to derive it).

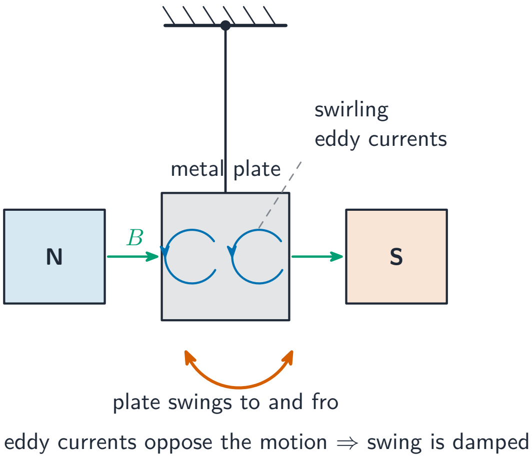


*Coils of wire spinning in a magnetic field: moving them past the field induces a current, the basis of a generator*


*A substation transformer: changing magnetic flux in coils induces the voltages that power the grid*

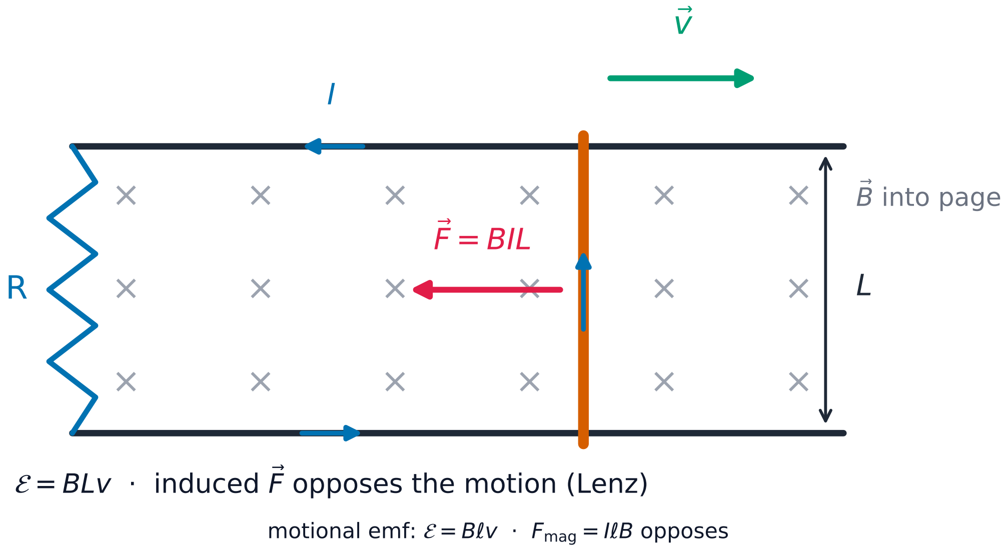
## Induced Currents and Magnetic Forces

Once an induced current flows, the external field exerts forces on it ($\vec{F} = \int I\, d\vec{l} \times \vec{B}$) – and by Lenz's law those forces always *resist the motion* that causes the induction. Only the segments of the loop actually inside the field feel a force, which can make a loop accelerate, rotate, or brake. This is the origin of **eddy currents** 涡电流 braking, and you can apply Newton's second law to a moving loop or rod like any other mechanics problem.




*Induced eddy currents oppose motion, quickly damping a metal plate swinging in a field*

The classic setup is a rod sliding on conducting rails connected by a resistor:




*A rod sliding on rails: the induced current feels a force opposing the motion*

**Worked example (the full chain).** Rails $L = 0.20$ m apart with resistance $R = 0.60\ \Omega$ sit in a 0.50 T field into the page. The rod is pushed at a constant 3.0 m/s. Then: $\varepsilon = BLv = 0.30$ V; $I = \varepsilon/R = 0.50$ A; the field pushes back on the rod with $F = BIL = 0.50(0.50)(0.20) = 0.050$ N. The pushing force does work at $P = Fv = 0.15$ W – exactly the $P = I^2R = 0.15$ W dissipated in the resistor. Mechanical work becomes electrical energy: that is a **generator** 发电机, and energy is conserved. Released with no push, the rod slows exponentially: $ma = -\dfrac{B^2L^2}{R}v$.

**Exam skill.** FRQs walk this exact chain: flux → emf → current → force → Newton's second law. Write each link separately and check the direction with Lenz's law at the end.

# Inductance

**Inductance** 电感 is a conductor's tendency to oppose a change in its own current: changing current changes its own flux, which self-induces an emf. From Faraday's law,

$$\varepsilon = -L\frac{dI}{dt}.$$

An **inductor** 电感器 is a circuit element built to have large inductance – usually a **solenoid** 螺线管, where geometry gives

$$L_{\text{sol}} = \frac{\mu_{\text{core}} N^2 A}{\ell},$$

with $N$ total turns, area $A$, length $\ell$, and the **magnetic permeability** 磁导率 of the core. (Straight wires are modelled as having zero inductance.) An inductor carrying current $I$ stores energy in its magnetic field:

$$U_L = \tfrac{1}{2} L I^2,$$

which can later be dissipated in a resistor or moved into a capacitor – conservation of energy applies as usual.

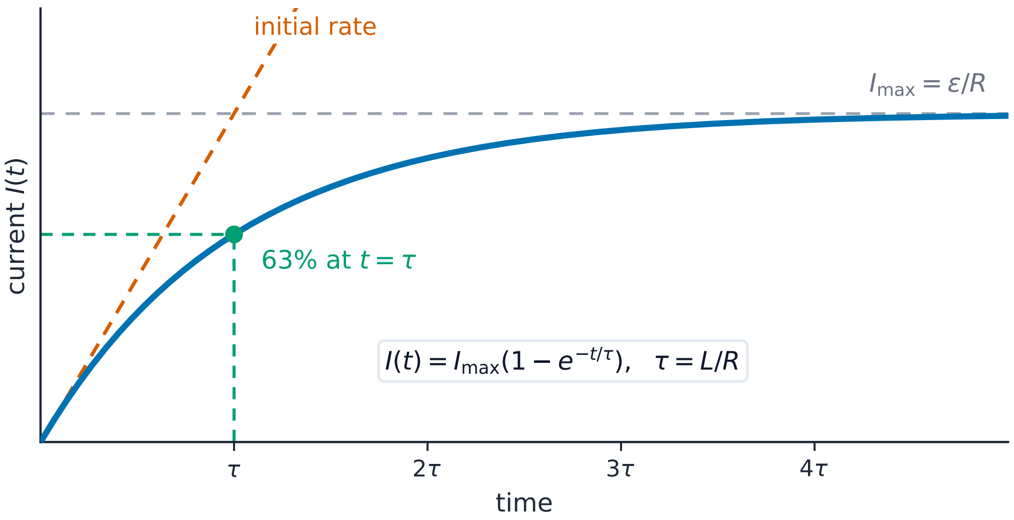
# Circuits with Resistors and Inductors

In an **RL circuit** RL 电路, Kirchhoff's loop rule for a battery $\varepsilon$, resistor $R$, and inductor $L$ in series gives a **differential equation** 微分方程:

$$\varepsilon = IR + L\frac{dI}{dt} \quad \Rightarrow \quad I(t) = \frac{\varepsilon}{R}\big(1 - e^{-t/\tau}\big), \qquad \tau = \frac{L}{R}.$$

The **time constant** 时间常数 $\tau$ sets the pace: after one $\tau$ a rising current reaches about 63% of its final value (a decaying one falls to 37%); it is also how long the change *would* take at the initial rate. Learn the two limits – they answer most conceptual questions:

- **Just after the switch closes** ($t = 0$): current cannot jump, so the inductor momentarily blocks it, self-inducing an emf equal and opposite to the applied potential difference.
- **Long after** ($t \gg \tau$): the current is steady, $dI/dt = 0$, and the inductor behaves as a plain wire – the exact opposite of a capacitor.




RL rise: $I = I_0(1 - e^{-t/\tau}) \cdot \tau = L/R$

*The current in an RL circuit rises exponentially, reaching 63% at one time constant*

**Worked example.** $\varepsilon = 12$ V, $R = 6.0\ \Omega$, $L = 3.0$ H: final current $\varepsilon/R = 2.0$ A, $\tau = L/R = 0.50$ s. At $t = 0.50$ s the current is $2.0(1 - e^{-1}) \approx 1.3$ A. At $t = 0$ the inductor's potential difference is the full 12 V; as $t \to \infty$ it falls to zero. Current, inductor voltage, and stored energy are all exponential in time.

# Circuits with Capacitors and Inductors

An **LC circuit** LC 电路 has no resistance, so nothing dissipates energy: it **oscillates** 振荡, sloshing energy between the capacitor's electric field and the inductor's magnetic field. The loop rule gives

$$\frac{d^2q}{dt^2} = -\frac{1}{LC}\,q,$$

the same equation as a mass on a spring – **simple harmonic motion** 简谐运动 with charge playing the role of displacement and

$$\omega = \frac{1}{\sqrt{LC}}.$$

The total energy $\dfrac{q^2}{2C} + \frac{1}{2}LI^2$ stays constant: all in the capacitor at maximum charge, all in the inductor at maximum current.

**Worked example.** $L = 2.0$ H, $C = 8.0$ $\mu$F: $\omega = \dfrac{1}{\sqrt{2.0(8.0 \times 10^{-6})}} = 250$ rad/s, period $T = 2\pi/\omega \approx 0.025$ s. If the capacitor starts charged to 12 V, then $U = \frac{1}{2}CV^2 = 5.8 \times 10^{-4}$ J, and the maximum current follows from $\frac{1}{2}LI_{\text{max}}^2 = U$: $I_{\text{max}} = \sqrt{2U/L} = 0.024$ A – **conservation of energy 能量守恒**, no calculus needed.

# Exam tips

- Compute **flux** $\Phi_B = \int \vec{B} \cdot d\vec{A}$ and get the induced EMF from **Faraday's law** $\varepsilon = -\frac{d\Phi_B}{dt}$.
- Use **Lenz's law** (the minus sign) to fix the direction: the induced current opposes the change in flux.
- Flux changes three ways — changing $B$, changing area, or changing angle — identify which and differentiate.
- For a rod of length $L$ moving at speed $v$, the motional EMF is $BLv$.
- An inductor stores energy $\frac{1}{2}LI^2$ and resists **changes** in current (RL time constant $\tau = L/R$).