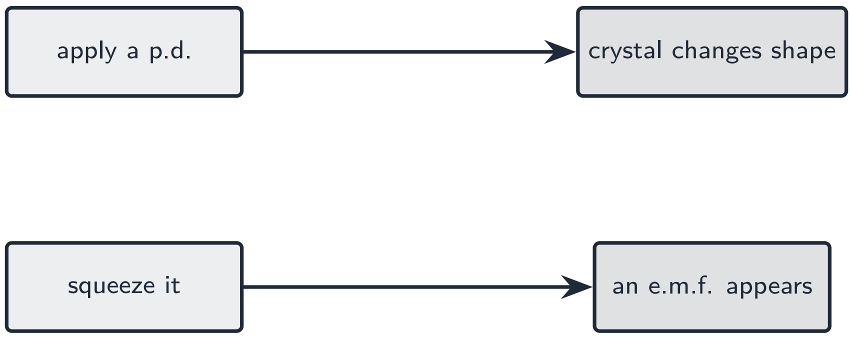
Medical physics

A-Level Physics

Ultrasound

Piezo-electric effect

A **piezo-electric** 压电 crystal changes shape a little when a p.d. is put across it, and the reverse: it makes an **electromotive force** 电动势 (e.m.f.) across itself when its shape is changed. Quartz and PZT are common examples. Two linked effects:



Applying a p.d. deforms the crystal; squeezing it makes a p.d.

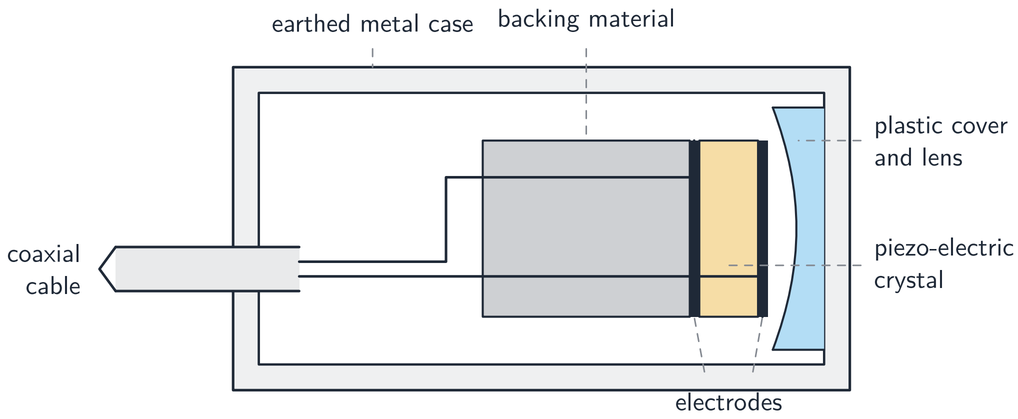
- **apply a p.d.** → the crystal changes shape (used to make vibrations).
- **change the shape** (a wave squeezes it) → an e.m.f. appears (used to detect vibrations).

Piezo-electric transducer

A **transducer** 换能器 uses this effect to both **make** and **detect ultrasound** 超声波.

- an alternating p.d. (a few MHz) makes the crystal vibrate at the same frequency, sending out **longitudinal** 纵波 waves above 20 kHz (1–10 MHz for medical imaging).
- the same crystal then detects: returning ultrasound makes it vibrate and produce an e.m.f.

So one transducer is both emitter and detector, switching between sending and listening.



A piezo-electric transducer both sends and detects ultrasound using a vibrating crystal

Pulse-echo imaging

To see inside the body (**pulse-echo 脉冲回波** imaging):

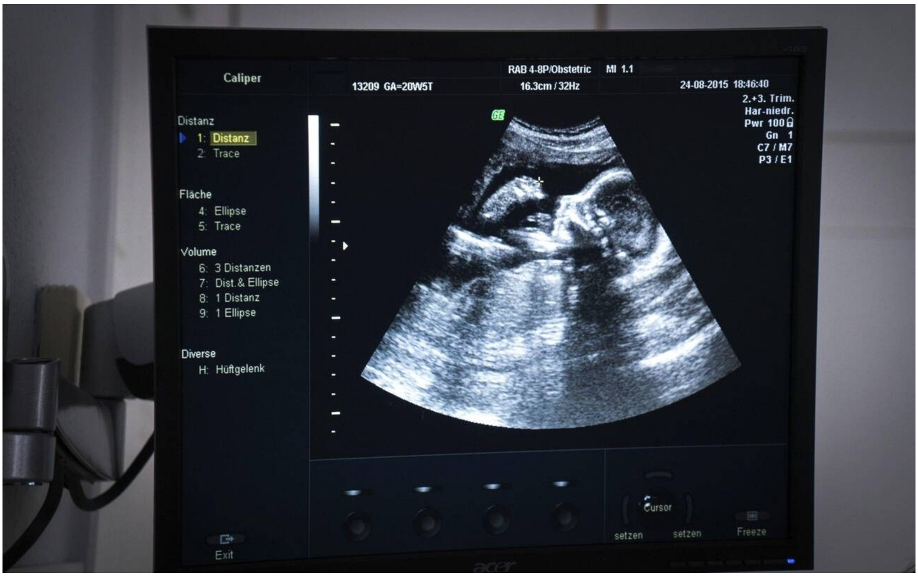
1. the transducer sends a **short pulse** into the body.
2. at each tissue boundary, part of the pulse is **reflected** and part goes on.
3. the transducer detects each reflected pulse.
4. the **time delay** gives the depth: $d = ct/2$ (there and back). The echo's amplitude gives the strength of the reflection.

Worked example. An ultrasound pulse returns to the transducer 60 μs after it was sent. The speed of sound in the tissue is 1500 m s^{-1}. Find the depth of the reflecting boundary.

The pulse travels there and back, so $d = \dfrac{ct}{2}$:

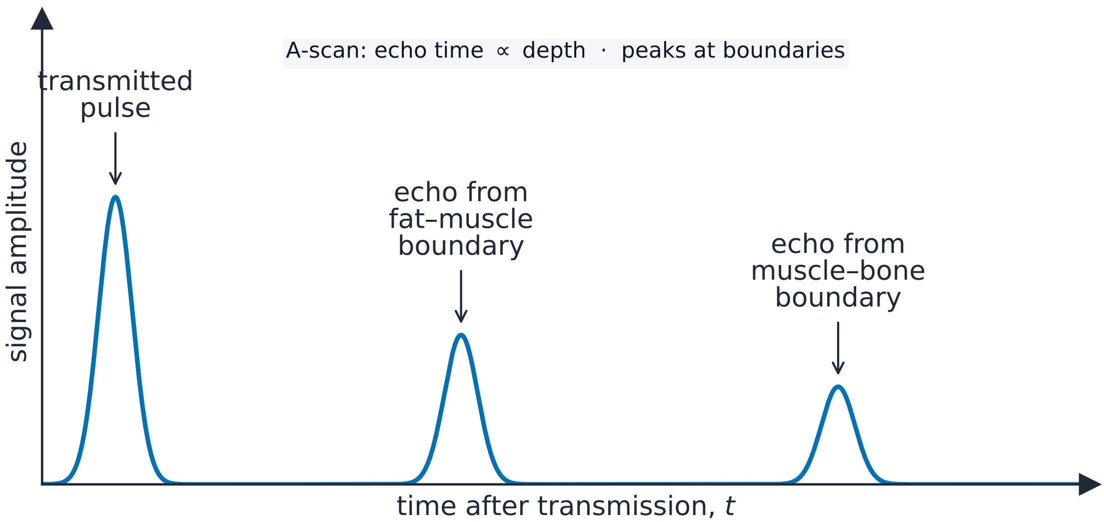
$$d = \frac{1500 \times 60 \times 10^{-6}}{2} = 0.045 \text{ m } (= 4.5 \text{ cm}).$$

1. sweeping across the body builds a 2-D image.



A real ultrasound image — the fan shape comes from the transducer sweeping across the body. Every bright speck is an echo from a boundary between tissues, and its depth was worked out from the echo's time delay, exactly as in the worked example above

A **coupling gel** 耦合剂 is put between the transducer and the skin to push out the air; without it almost all the ultrasound would reflect at the skin–air boundary and never enter the body.



An A-scan shows the transmitted pulse and the echoes from each tissue boundary

Specific acoustic impedance

The **specific acoustic impedance** 声阻抗 of a medium is

$$Z = \rho c,$$

where ρ is the **density** 密度 and c the speed of sound. Unit: kg m^{-2} s^{-1}. Bone has large Z; air has small Z; soft tissue is in between.

Worked example. Find the specific acoustic impedance of soft tissue. (Density 1060 kg m^{-3}, speed of sound 1540 m s^{-1}.)

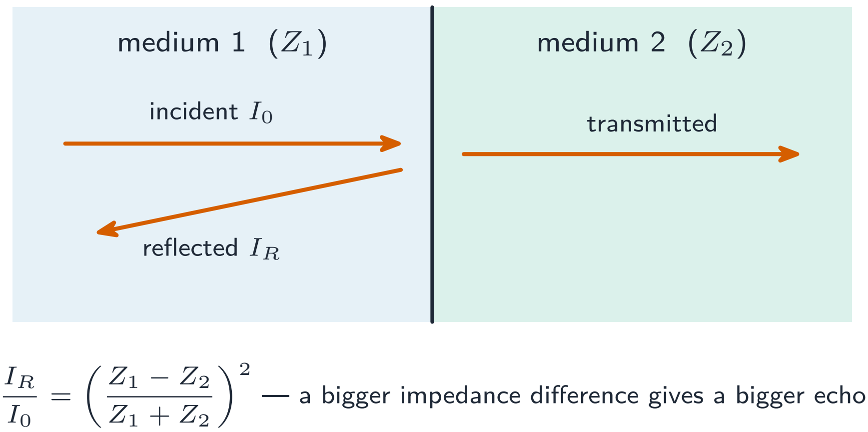
$$Z = \rho c = 1060 \times 1540 \approx 1.6 \times 10^6 \text{ kg m}^{-2} \text{ s}^{-1}.$$

Reflection at a boundary

At a boundary between media of impedance Z_1 and Z_2, the **intensity reflection coefficient** 强度反射系数 (fraction reflected) is

$$\frac{I_\mathrm{R}}{I_0} = \left(\frac{Z_1 - Z_2}{Z_1 + Z_2}\right)^2 .$$

- very different impedances: almost all is reflected (skin/air — hence the gel).
- very similar impedances: almost nothing is reflected, so the boundary cannot be seen.
- best for imaging: different enough to give an echo, but not so different that nothing passes on.



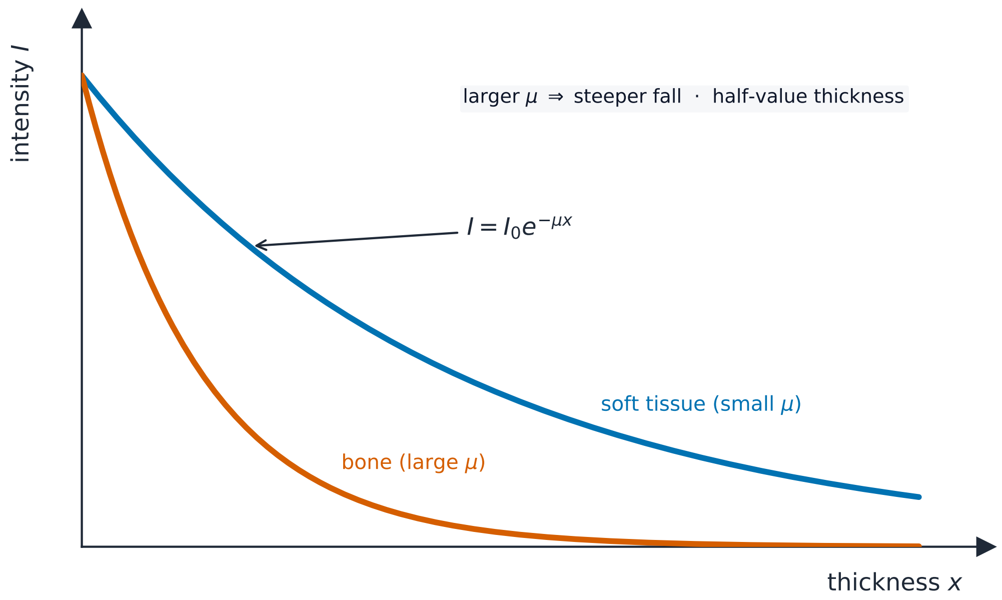
At a boundary, part of the pulse reflects and part transmits — a bigger impedance difference gives a bigger echo

Attenuation

As ultrasound goes through tissue, its intensity falls with distance:

$$I = I_0 e^{-\mu x},$$

where μ is the **attenuation coefficient** 衰减系数 (unit m^{-1}). The same form applies to X-rays.



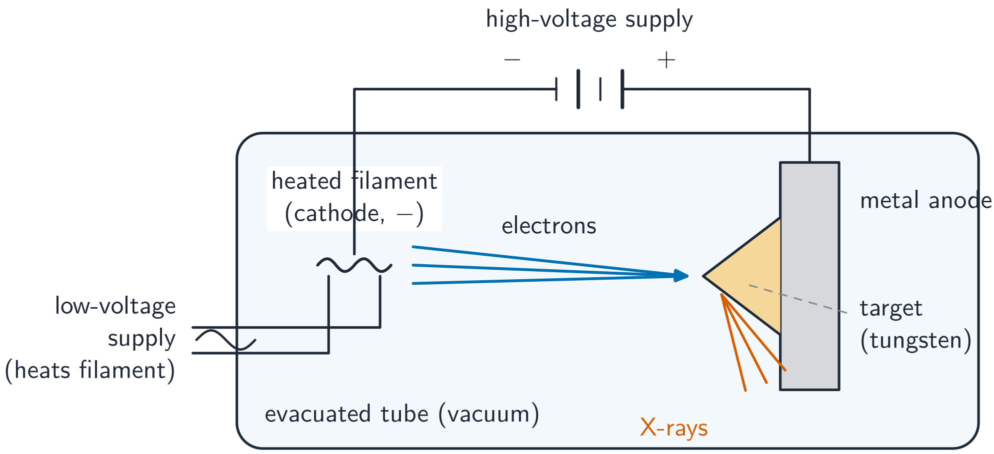
Intensity falls exponentially with thickness; a larger attenuation coefficient (bone) falls off faster

X-rays

Production

X-rays come from an **X-ray tube** X 射线管:

1. a heated **cathode** 阴极 emits electrons by **thermionic emission** 热电子发射.
2. a high p.d. (tens to hundreds of kV) accelerates the electrons across a **vacuum** 真空 to a metal **target** 靶 (the **anode** 阳极, often **tungsten** 钨).
3. the electrons hit the target and slow sharply. Most of their **kinetic energy** 动能 becomes heat; a small part is emitted as X-ray **photons** 光子 (**Bremsstrahlung** 韧致辐射, "braking radiation"). Some electrons knock out inner electrons of the metal atoms, and the refilling emits **characteristic** 特征 X-ray lines.



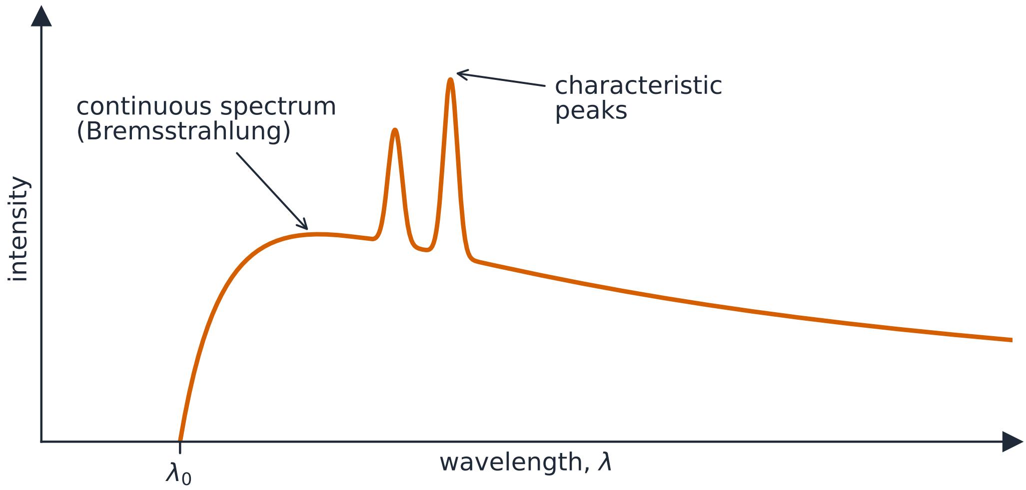
In an X-ray tube electrons from the heated cathode are accelerated onto a metal target anode

Minimum wavelength

The most energy one X-ray photon can have is the full kinetic energy of one accelerated electron, lost in a single event. For accelerating p.d. V, the KE is eV, so

$$hf_{\text{max}} = eV, \qquad \lambda_{\text{min}} = \frac{hc}{eV}.$$

This is the short-wavelength cut-off. The continuous Bremsstrahlung spectrum tails off above λ_{min}, with sharp characteristic peaks set by the target metal.



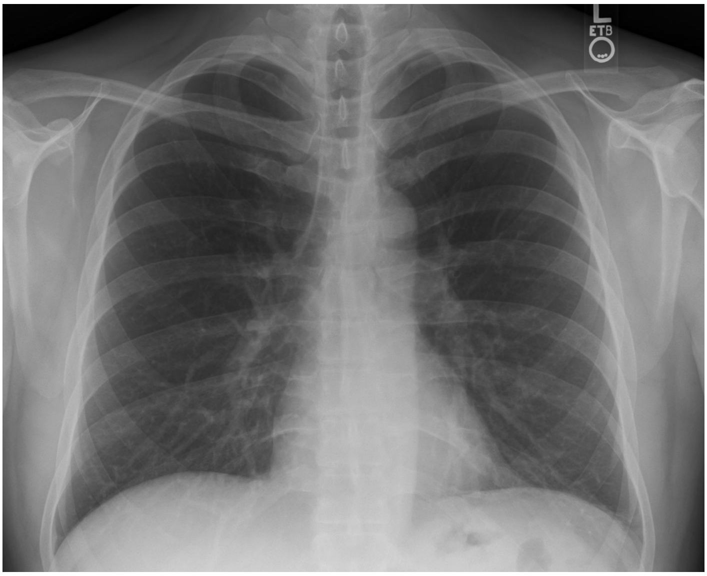
X-ray: continuous bremsstrahlung + characteristic peaks

A typical X-ray spectrum — a continuous Bremsstrahlung curve cut off at λ_0, with sharp characteristic peaks

Imaging with X-rays

X-rays pass through the patient onto a detector. Tissues that **attenuate** 衰减 more (bone, high Z) cast a stronger shadow and look lighter; tissues that attenuate less (soft tissue, lung) look darker.

The **contrast** 对比度 is the difference in attenuation between tissues. A **contrast medium** 造影剂 (e.g. a barium meal) can be given to make soft tissues stand out.



A real chest X-ray: dense bone absorbs more X-rays and looks white; the air-filled lungs let X-rays through and look dark

Attenuation law

$$I = I_0 e^{-\mu x}.$$

Higher-energy X-rays penetrate further (smaller μ); bone has a much larger μ than soft tissue. To find the thickness for a given fraction, take logs: $x = \dfrac{1}{\mu}\ln\dfrac{I_0}{I}$. The **half-value thickness** 半值厚度 $x_{1/2} = \ln 2/\mu$ halves the intensity (like half-life in decay).

Worked example. X-rays pass through 3.0 cm of tissue with attenuation coefficient $\mu = 40\ \text{m}^{-1}$. Find the fraction of the intensity that gets through.

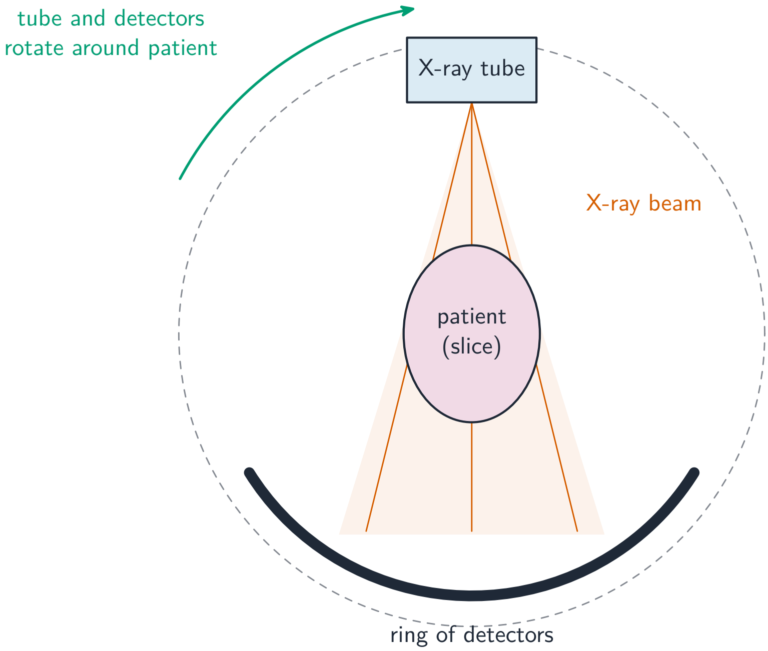
$$\frac{I}{I_0} = e^{-\mu x} = e^{-40\times 0.030} = e^{-1.2} \approx 0.30.$$

Computed tomography (CT)

A **computed tomography** 计算机断层扫描 (CT) scan builds a 3-D image:

1. the tube and detectors rotate around the patient, taking many images of one thin slice from **different angles**.
2. a computer combines these into a 2-D cross-section of the slice.
3. the patient is moved along, and the next slice is imaged.
4. the slices are stacked into a 3-D image.

CT shows far more than a single X-ray, because overlapping soft tissues are separated by the many-angle reconstruction.



In a CT scan the X-ray tube and detectors rotate around the patient to image a slice from many angles

PET scanning

Tracer

A **tracer** 示踪剂 is a substance with radioactive nuclei put into the body. It is taken up more by the tissue being studied (e.g. a tumour takes up more glucose-tagged tracer due to its high **metabolism** 代谢). Its decay is detected from outside.

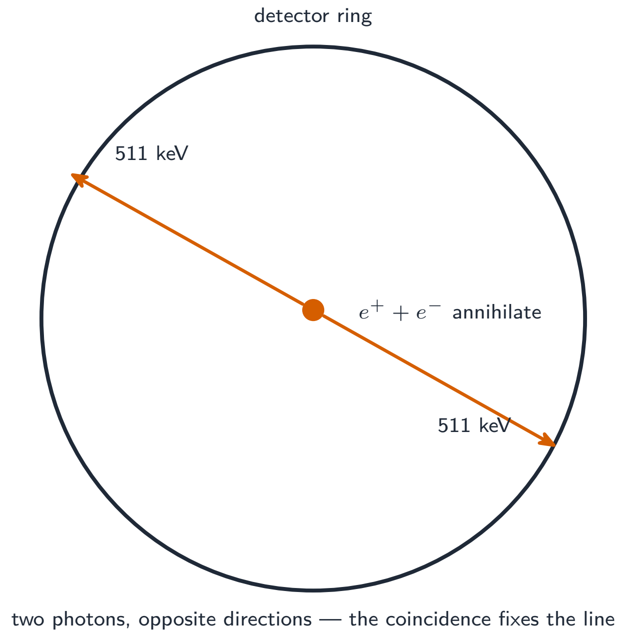
In **positron emission tomography** 正电子发射断层扫描 (PET), the tracer is a β^+

emitter — it gives out a **positron** 正电子. A common one is fluorine-18 on a glucose analogue (FDG).

Annihilation

When a particle meets its **antiparticle** 反粒子 they **annihilate** 湮灭: their mass turns into electromagnetic energy. In PET:

- a positron travels a few mm before meeting an **electron** 电子.
- they annihilate. Energy and **momentum** 动量 are conserved.
- since the total momentum is about zero, two photons are produced going in **opposite directions**, each 511 keV ($= m_e c^2$).



PET: annihilation gives two 511 keV photons in opposite directions; a coincidence fixes the line

Energy of the annihilation photons

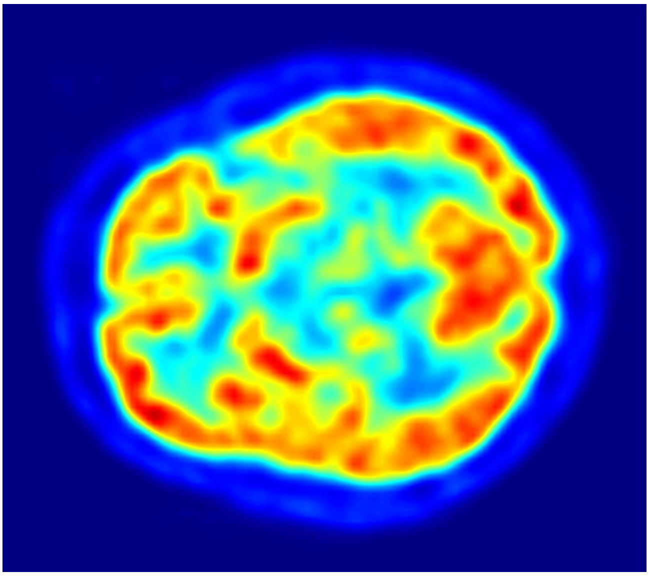
By energy conservation, the total photon energy equals the pair's rest **energy** 能量:

$$2hf = 2m_e c^2, \qquad hf = m_e c^2.$$

Each photon has $hf = m_e c^2 \approx 8.2 \times 10^{-14}$ J ≈ 0.51 MeV, with $\lambda \approx 2.4 \times 10^{-12}$ m.

Reconstructing the image

The two photons leave the body in opposite directions and hit **detector rings** around the patient. Recording the **two simultaneous arrivals** (a “coincidence”) fixes the line the annihilation happened on. Many coincidences from many angles let the computer build a 3-D map of the tracer — showing tissues with high metabolic activity. Comparing the two **arrival times** can refine the position along that line (time-of-flight PET).



A finished PET image of the brain: warm colours (red, yellow) mark where the tracer collected – the most active tissue

Exam tips

- **Ultrasound** reflects at boundaries (acoustic impedance); a coupling gel reduces reflection at the skin.
- **X-rays** are attenuated as $I = I_0 e^{-\mu x}$; contrast media (barium, iodine) absorb strongly.
- **PET** uses a positron emitter; annihilation gives two γ-rays in opposite directions, located by the detector ring.