Alternating currents

A-Level Physics

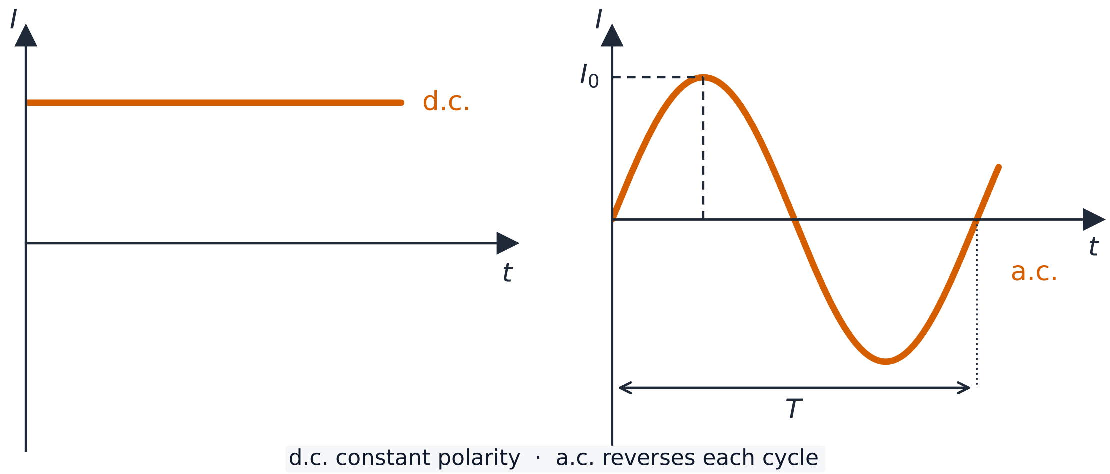
Alternating current basics

A substation's transformers step alternating voltage up or down.

An **alternating current** 交流电 (a.c.) keeps reversing direction. Mains supply is sinusoidal a.c.: I or V follows a sine wave in time:

$$I = I_0 \sin(\omega t), \qquad V = V_0 \sin(\omega t).$$

(For a purely resistive load the **voltage** 电压 and **current** 电流 are in phase, which is the case in this syllabus.)



A steady direct current compared with a sinusoidal alternating current of peak I_0 and period T

Key terms

- **period** 周期 T — the time for one full cycle. Unit: s.
- **frequency** 频率 f — cycles per second; $f = 1/T$. Mains is often 50 Hz or 60 Hz.
- **angular frequency** 角频率 $\omega = 2\pi f = 2\pi/T$.
- **peak value** 峰值 I_0 or V_0 — the largest value in a cycle (also called the amplitude).
- **peak-to-peak value** 峰峰值 $2I_0$ — from $+I_0$ to $-I_0$. Useful when reading an oscilloscope.

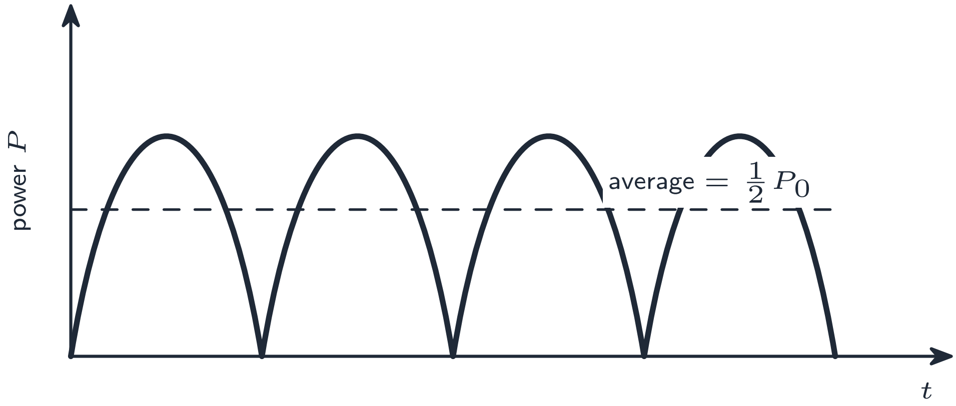
Reading a CRO trace

Same as for any wave (Topic 7), using a **cathode-ray oscilloscope** 示波器:

- horizontal divisions × **time-base** 时基 → period T, so $f = 1/T$.
- vertical divisions × y-gain → peak voltage V_0 (measure centre to peak, or peak-to-peak then halve).

Power delivered to a resistor

For a resistive load R, the instant **power** 功率 is $P(t) = I(t)^2 R$. With $I = I_0 \sin(\omega t)$:



The power in a resistor pulses; its average is half the peak

$$P(t) = I_0^2 R \sin^2(\omega t).$$

This is always positive, with peak $I_0^2 R$ and minimum zero, oscillating at twice the frequency of I. The mean of $\sin^2(\omega t)$ over a cycle is $\frac{1}{2}$, so the **average power** is

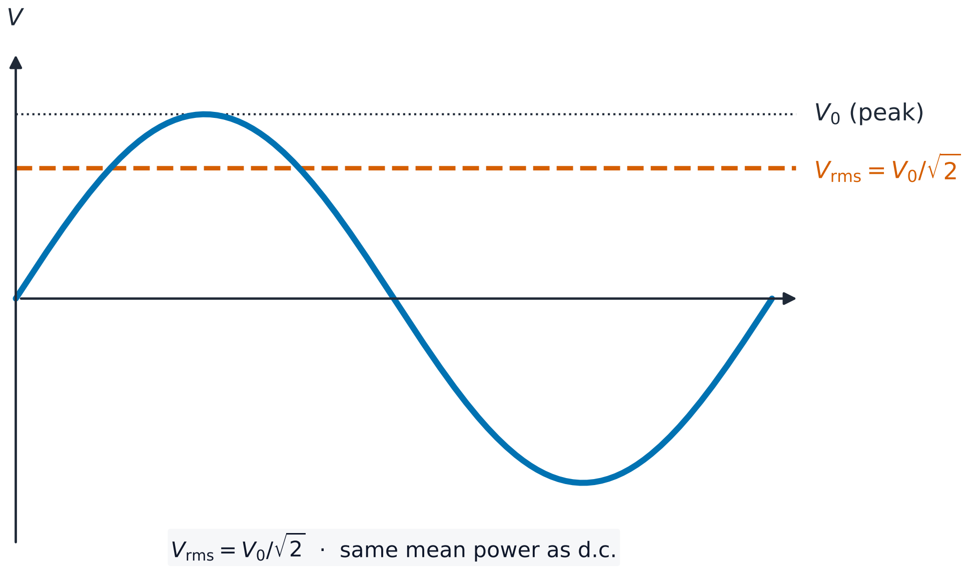
$$\langle P \rangle = \tfrac{1}{2} I_0^2 R = \tfrac{1}{2} P_{\text{peak}}.$$

Average a.c. power in a resistor is **half the peak power**.

Root-mean-square (r.m.s.) values

The r.m.s. current $I_{\text{r.m.s.}}$ is the **steady direct current** that would give the same average power in the same **resistance** 电阻 R. From $\langle P \rangle = I_{\text{r.m.s.}}^2 R = \frac{1}{2} I_0^2 R$:

$$I_{\text{r.m.s.}} = \frac{I_0}{\sqrt{2}}, \qquad V_{\text{r.m.s.}} = \frac{V_0}{\sqrt{2}}.$$



The r.m.s. value is the steady d.c. level ($V_0/\sqrt{2}$) that delivers the same average power as the a.c.

Worked example. An a.c. supply has a peak voltage of 12 V. Find its r.m.s. voltage.

$$V_{\text{r.m.s.}} = \frac{V_0}{\sqrt{2}} = \frac{12}{\sqrt{2}} \approx 8.5 \text{ V}.$$

The $\sqrt{2}$ comes from the name **root-mean-square** 均方根: $I_{\text{r.m.s.}} = \sqrt{\langle I^2 \rangle}$ and $\langle \sin^2 \rangle = \frac{1}{2}$. (Only the sinusoidal case is needed.)

Why r.m.s. matters

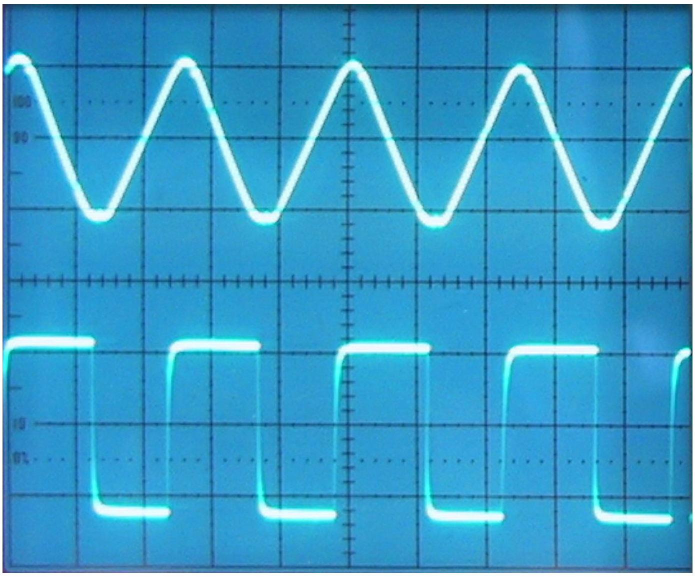
Quoted a.c. values are r.m.s. values. "230 V mains" means $V_{\text{r.m.s.}} = 230$ V, with peak $V_0 = 230\sqrt{2} \approx 325$ V. Components must be rated for the peak, not the r.m.s. Average power then takes the d.c. form:

$$\langle P \rangle = I_{\text{r.m.s.}}^2 R = V_{\text{r.m.s.}}^2 / R = V_{\text{r.m.s.}} I_{\text{r.m.s.}}.$$

Worked example. A heater of resistance 50 Ω is connected to the 230 V r.m.s. mains. Find the r.m.s. current and the average power dissipated.

$$I_{\text{r.m.s.}} = \frac{V_{\text{r.m.s.}}}{R} = \frac{230}{50} = 4.6 \text{ A}, \qquad \langle P \rangle = V_{\text{r.m.s.}} I_{\text{r.m.s.}} = 230 \times 4.6 \approx 1.1 \times 10^3 \text{ W}.$$

Rectification



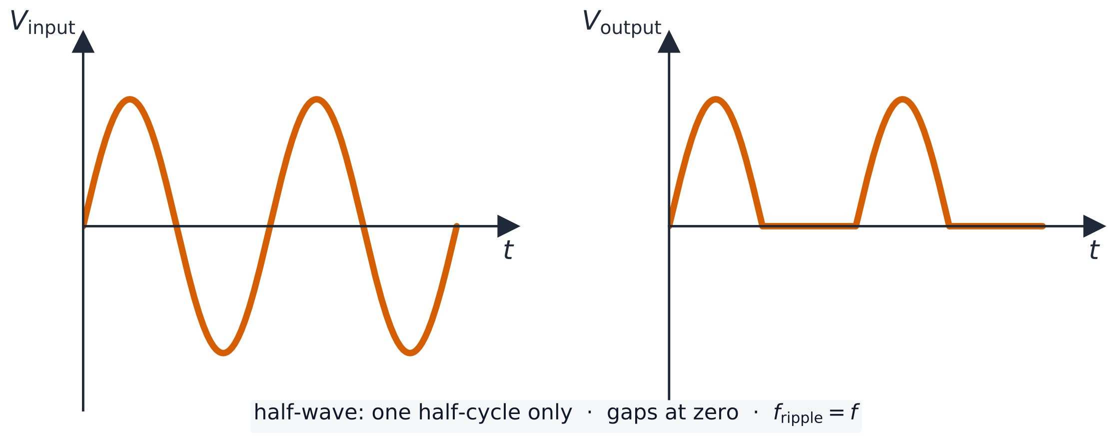
An oscilloscope shows how a voltage varies with time.

Rectification 整流 turns an alternating voltage into a one-direction (d.c.-like) voltage, using **diodes** 二极管 (which conduct in only one direction).

Half-wave rectification

A **single diode** in series with the load passes only the positive half of each cycle; in the negative half the diode is **reverse-biased** 反向偏置 and no current flows. This is **half-wave rectification** 半波整流.

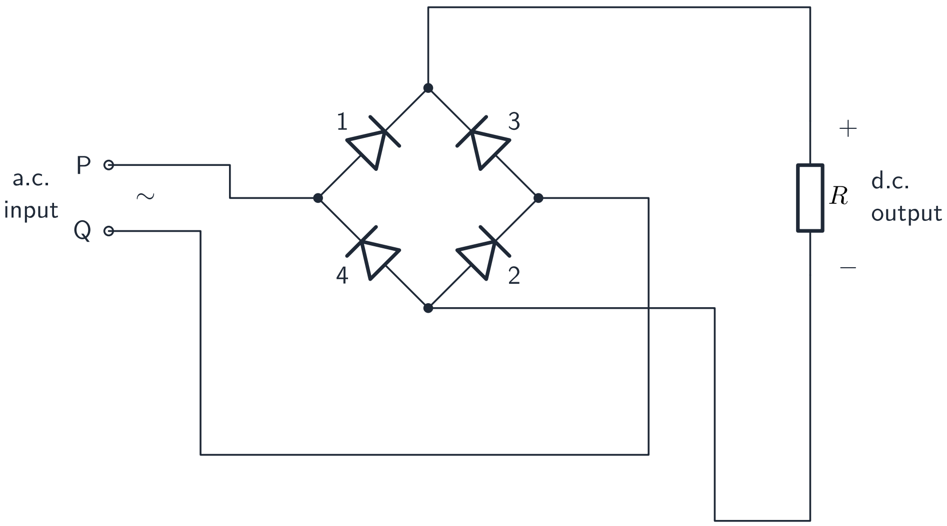
Output: positive half-waves with flat zero gaps. The mean output is $V_0/\pi \approx 0.32V_0$. Drawback: half the input is wasted and the output is very uneven.



In half-wave rectification a single diode passes only the positive half-cycles

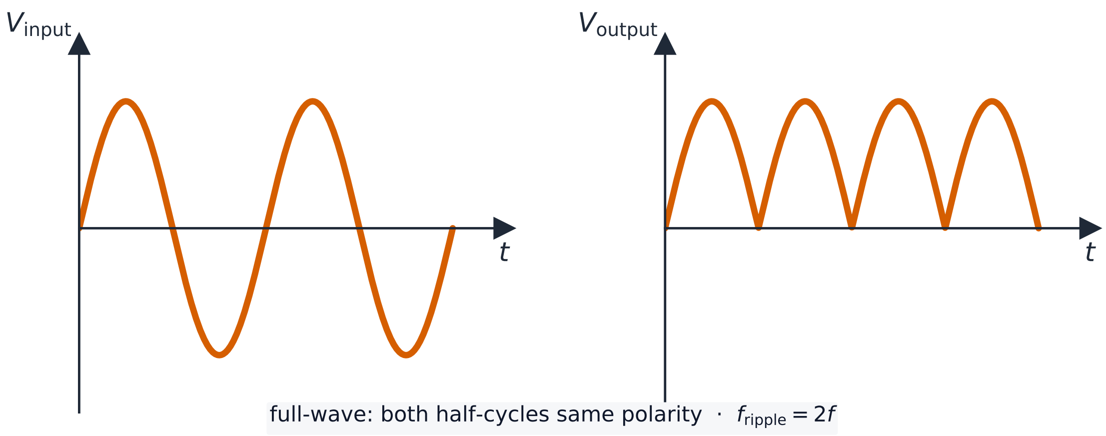
Full-wave rectification (bridge rectifier)

A **bridge rectifier** 桥式整流器 uses **four diodes** arranged so the current through the load always flows the same way, whichever a.c. terminal is positive — **full-wave rectification** 全波整流. On each half-cycle a different pair of diodes conducts, but the load always sees the same direction.



A four-diode bridge sends the load current the same way whichever a.c. terminal is positive

Output: a continuous run of positive half-waves (no gaps), at **twice** the input frequency. The mean output is $2V_0/\pi \approx 0.64V_0$ — double the half-wave value. It uses all the input and is smoother and easier to filter.



In full-wave rectification every half-cycle is used, giving a continuous run of positive humps

Drawing the diagrams

- half-wave: a.c. source — single diode — load R, in series.
- full-wave bridge: four diodes as the arms of a "diamond"; the a.c. input goes to one pair of opposite corners, the load R across the other pair. The diode directions make the load terminals keep the same polarity for either input polarity.

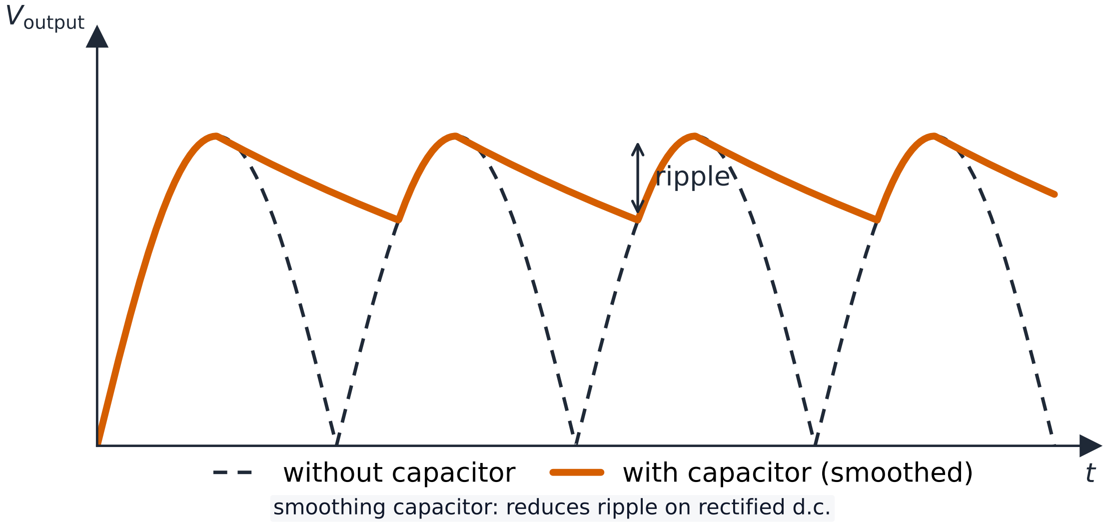
Smoothing with a capacitor

A rectifier's output is still bumpy. To smooth it, put a **capacitor 电容器 C in parallel with the load R.**

How it works

- on the **rising part** of each pulse, the capacitor charges up to near the peak.
- on the **falling part** (and any gap), the diodes are reverse-biased, so the capacitor **discharges through the load**, keeping current flowing. The voltage falls with **time constant 时间常数 RC** (Topic 19).
- at the next peak, the capacitor charges again, and the cycle repeats.

The output now sits near the peak with small dips. The size of the dips is the **ripple** 纹波 (this whole step is called **smoothing 平滑**).



A capacitor across the load smooths the rectified output, leaving only a small ripple

What reduces the ripple

- **larger** C → more stored charge → smaller dip between peaks → smaller ripple.
- **larger** R → smaller load current → slower discharge → smaller ripple.
- **higher rectified frequency** (full-wave is twice the input) → less time to discharge between peaks → smaller ripple.

In short, a large RC compared with the time between peaks gives a smoother output.

Purpose in summary

The smoothing capacitor reduces the ripple, giving a steadier d.c. voltage suitable for sensitive electronics.

Exam tips

- $I_{rms} = I_0/\sqrt{2}$ for a sinusoidal current; use r.m.s. values for power ($P = I_{rms}^2 R$).
- Mean power in a resistor $= \frac{1}{2}$ of the peak power.
- Explain **rectification** (diode or bridge) and how a **smoothing capacitor** reduces the ripple.